Optimal Central Bank Swap Line Policy[*]

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This paper studies the optimal central bank swap line policy. Due to a pecuniary externality, there is a trade-off between ex-ante and ex-post efficiency of the swap line policy. During financial crises, the swap line policy lowers CIP deviations and prevents fire sale of source currency assets, beneficial to both recipient and source country. However, it makes recipient banks to overborrow ex-ante, sowing the seeds of financial crises. From a global welfare point of view, the ex-post efficient policy is more lenient than the ex-ante efficient policy, which implies time inconsistency. The policy mix with macroprudential policies can correct the overborrowing problem and resolve time inconsistency. Moreover, policy coordination of a cooperative Ramsey problem obtains undersupply (oversupply) of source currency provision under a realistic condition when the source country has higher (lower) bargaining power.

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1. Introduction

During the global financial crisis (GFC) and COVID-19 crisis, central bank swap lines were used as liquidity facilities to stabilize international financial markets. The source central bank, usually the Federal Reserve, provides its own currency to recipient central banks, playing a role as an international lender of last resort. This swap line policy is known to prevent the widening of deviations from covered interest rate parity (CIP) and to lower synthetic dollar funding costs (Bahaj and Reis, 2022). As non-US financial intermediaries rely more on synthetic dollar funding during financial distress periods, the swap line policy alleviates strains in dollar funding markets.

However, potential side-effects of the swap line policy are still understudied. As other domestic liquidity provision policies, the swap line policy can also result in overborrowing problem. Expecting swap lines and the following decline in dollar funding costs, financial intermediaries in the recipient country accumulate excessive amount of borrowing ex-ante. As pointed out by previous literature such as Bianchi (2011), this overborrowing problem can lead to higher probability of financial crises in the recipient country. However, this is not the whole story; the build-up of risk affects not only the recipient country but also the source country. Through international asset markets, fire sale of source currency assets by recipient banks can erode balance sheets of source country banks, resulting in the contamination of financial crises.

Considering the trade-off of liquidity provision in international financial markets, this paper studies the optimal central bank swap line policy. For this purpose, following Jeanne and Korinek (2020), I construct a three-period model with two countries: the US and the EU. The US is the source country while the EU is the recipient country. In each country, there are two agents: bankers and depositors. Finally, there is only one asset in this model, which is the US asset that conveys payoffs denominated in US dollar.

Banks in both countries invest in US assets. In addition, US banks supply synthetic dollar funding by arbitraging CIP deviations. Following Ivashina, Scharfstein, and Stein (2015), CIP deviations emerge since US banks are subject to a margin requirement for the CIP arbitrage. EU banks demand synthetic dollar funding when they cannot directly borrow US dollar. Since their US asset holdings are denominated in US dollar, currency matching assumption requires them to use FX swap to convert local currency funding into synthetic dollar funding. CIP deviations are then intermediation fees that they need to pay for using FX swap.

Both banks are subject to collateral constraints on their deposits. Since they do not take effects of their choices on the collateral value into account, there is a pecuniary externality. The pecuniary externality amplifies inefficiency since there are collateral constraints as financial frictions. The economy is defined to be under normal (crisis) regime when collateral constraints are binding (non-binding).

Based on this model, I study the trade-off between the ex-post efficiency and the ex-ante efficiency of the swap line policy. When there is a crisis, banks desire to sell their assets, resulting in a plunge in the US asset price and a decline in investment. The swap line policy prevents fire sales of US assets by providing US dollar liquidity to EU banks faced with shortage of dollars. As EU banks are replenished with dollar liquidity, less demand for synthetic dollar funding leads to lower CIP deviations while the US asset price is stabilized. This is beneficial not only for the EU but also for the US since the asset market is integrated. These benefits are amplified by the pecuniary externality since the swap line policy relaxes collateral constraints. The ex-post efficient policy equates marginal benefits for the world economy with marginal change in deadweight loss of supplying dollar liquidity.

However, the swap line policy leads to an ex-ante overborrowing problem. Since EU banks know that dollar liquidity will be injected in their balance sheets through swap lines, they have more incentive to accumulate larger amount of liabilities ex-ante. However, larger deposits lead to higher probability of crises. Since there is a pecuniary externality, amplification of crises is not considered by banks. This implies that there is an overborrowing and the economy fails to achieve constrained efficiency. Taking the overborrowing problem into account, the ex-ante efficient policy, or swap line policy under commitment, provides less dollar liquidity than the ex-post efficient policy.

Next, I show that a policy mix with macroprudential tools can fix the time inconsistency. Tax on deposits can correct the pecuniary externality by equating social marginal costs of deposits with private marginal costs, and achieves constrained efficiency. Then, the ex-post efficient swap line policy becomes equivalent to the ex-ante efficient swap line policy. This implies that the swap line policy is time consistent. Intuitively, one policy instrument, central bank swap lines, is not enough to accomplish two objectives: the ex-ante efficiency and the ex-post efficiency. We need an additional policy instrument, which is the macroprudential policy.

Finally, I investigate the issue of policy coordination between separate governments. The existence of a global Ramsey planner who maximizes the world welfare function may be unrealistic. Thus, I construct a cooperative Ramsey problem defined as maximizing a weighted average of the US and the EU welfare function subject to implementability conditions (Chari, Nicolini, and Teles, 2023). Here, the weight on each welfare function is the bargaining power of each country. Under a condition that US banks have more US dollar liquidity than EU banks, I show that the liquidity provision through swap lines is undersupplied when the US has higher bargaining power, and vice versa.

Related Literature

With some earlier papers (Baba and Packer (2009a,b)), there are recent papers on effects of the swap line policy on CIP deviations and asset prices such as Bahaj and Reis (2022) and Kekre and Lenel (2023). Exploiting high-frequency data, these papers show that the swap line policy narrows CIP deviations, increases equity prices, and results in larger capital inflows into source-currency assets. However, all these papers conduct positive analysis on central bank swap lines, while the welfare analysis on this policy is absent. This paper focuses on the optimality of the swap line policy instead, and thus contributes to this literature by providing a normative analysis.

On the other hand, this paper extends the literature on lender of last resort and liquidity provision policy to an international setting. In this regard, this paper is related to research on pecuniary externalities and financial crises (Lorenzoni, 2008; Jeanne and Korinek, 2020; Schmitt-Grohé and Uribe, 2021 for instance). In particular, I extend Jeanne and Korinek (2020) to a two-country model and embed an FX swap market with a margin constraint following Ivashina, Scharfstein, and Stein (2015). Papers on the aggregate demand externality (Schmitt-Grohé and Uribe, 2016; Bianchi, 2016; Farhi and Werning, 2016; Korinek and Simsek, 2016) or collective moral hazard (Farhi and Tirole, 2012) analyze the related issue from different angles.

This paper is organized as follows. In section 2, I present a brief overview of central bank swap line policy. Section 3 presents a three-period two-country model with an FX swap market. Based on this model, I investigate the optimal swap line policy in section 4. Section 5 concludes.

2. Overview of Central Bank Swap Lines

Before moving onto the model, I briefly summarize the basic operation of central bank swap lines. For more institutional details on central bank swap lines, refer to Bahaj and Reis (2023).

Central bank swap lines are collateralized international liquidity facilities that a source central bank lends its currency to a recipient central bank. The recipient central bank pledges its currency as a collateral, so swap lines take the form of exchanging currencies of the two central banks engaged in this operation. The amount of collateral is determined by the market exchange rate. Then, the recipient central bank lends the source currency to banks in its jurisdiction, usually collateralized as domestic liquidity facilities. Figure 1 describes cash flows of central bank swap lines.

The interest rate of central bank swap lines is a swap spread over a risk-free rate, usually an overnight index swap (OIS) rate. According to Bahaj and Reis (2022), the swap

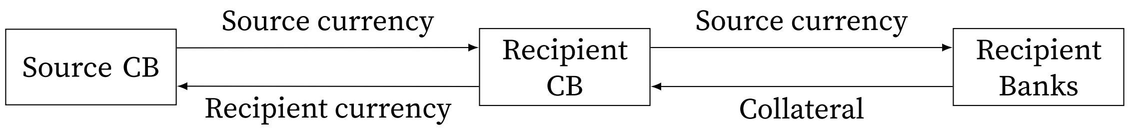

FIGURE 1. Cash Flows of Central Bank Swap Lines

line spread ss works as an upper bound on CIP deviations as

$$cid \equiv r^{OIS*} + s - f - r^{OIS} \leq ss + (r^{p*} - r^{IOER*})$$

where cid is an OIS-based CIP deviation for OIS rates r^{OIS} and r^{OIS*} of the source and the recipient currency, log spot exchange rate s, and log forward exchange rate f.[1] r^{p*} and r^{IOER*} are the policy rate and the interest rate on excess reserves (IOER) of the recipient currency respectively. This inequality holds due to a no-arbitrage condition. First, the cost of source currency funding through swap lines is $r^{OIS} + ss$, which is a fixed rate. On the other hand, the riskless return on this funding is $s - f + r^{IOER*} + r^{OIS*} - r^{p*}$. The source currency funding is converted to the recipient currency with the exchange rate risk hedged at the rate of $s - f$, and the riskless return on this recipient currency is r^{IOER*}. However, since r^{IOER*} is a floating rate, it is converted to a fixed rate with the return rate of $r^{OIS*} - r^{p*}$. No-arbitrage requires that $r^{OIS} + ss \geq s - f + r^{IOER*} + r^{OIS*} - r^{p*}$, which is equivalent to $cid \leq ss + (r^{p*} - r^{IOER*})$.[2] This inequality implies that the swap line policy is an international version of the domestic discount window of which the discount rate imposes a ceiling on the federal funds rate.

Maturities of swap lines are usually overnight, 1-week, and 1-month, with the maximum of 3-month. At the maturity, the source central bank and the recipient central bank exchange currencies back at the same exchange rate as the initial transaction.

The limit on lending through swap lines is determined by agreements between the two central banks. The exceptions are standing swap lines, which offer unlimited loans to recipient central banks in Canada, UK, EU, Japan, and Switzerland.

Figure 2 displays drawings of Fed swap lines from Dec 20 2007 to Jun 21 2024. For each settlement date, transactions with various counterparty central banks and maturities are aggregated. We can see that there were large amount of operations during the GFC, European sovereign debt crises, and COVID-19 crises.

[1] The spot and the forward exchange rate are in units of the recipient currency per source currency.

[2] The no-arbitrage condition is one-sided since it is impossible for recipient banks to lend source currency to the source central bank through swap lines.

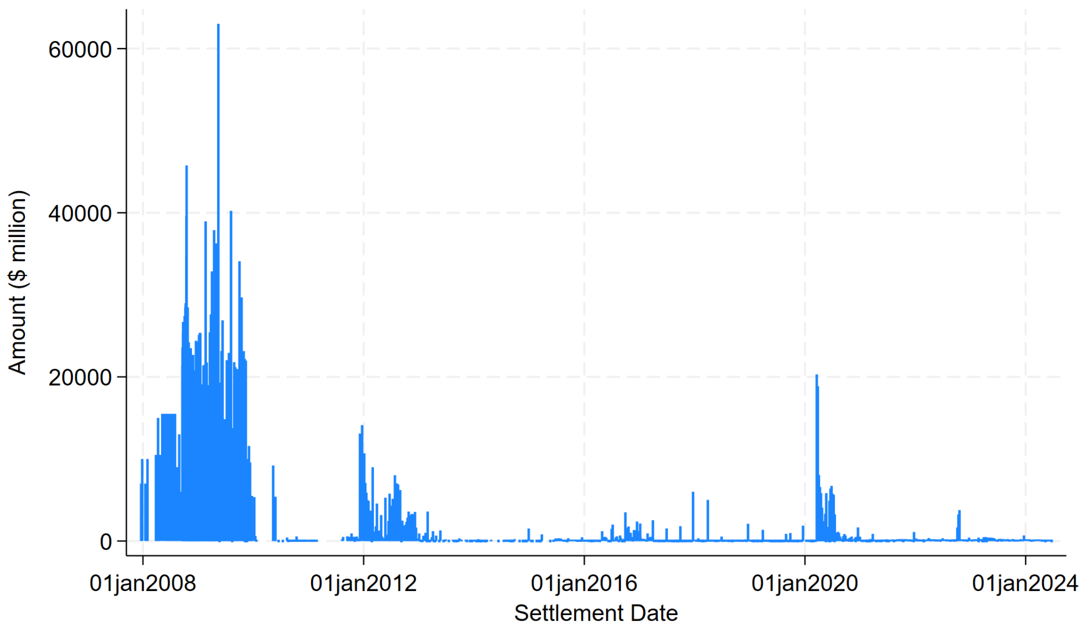

FIGURE 2. Drawings of Fed Swap Lines

Note. This figure shows transaction value of Fed swap line operations from Dec 20 2007 to Jun 21 2024. For each settlement date, transactions with various counterparty central banks and maturities are aggregated. Small value transactions for testing operations are excluded. Data for this figure can be obtained from the Federal Reserve Bank of New York.

Simplifying Assumptions

In order to simplify the model, the following assumptions for the operation of central bank swap lines are imposed:

i. Direct lending to recipient banks: The main economic agents in this model are the provider and users of the liquidity: the source central bank and recipient banks. The recipient central bank is the "intermediary" of swap lines, imposing covenants, monitoring recipient banks, and absorbing losses from defaults of banks. Since this paper focuses on pecuniary externality and the baseline model assumes no default of banks, I abstract the recipient central bank away.
ii. No distinction between interest rates: $r^{IOER*} = r^{p*}$, so $cid \leq ss$
iii. No distinction between maturities: one-period lending

3. Model

Extending Jeanne and Korinek (2020) to an international setting, I study a 3-period model with two countries: the US and the EU. The US is the country with the source currency (USD, $) while the EU is the country with the recipient currency (Euro, €). Bankers in both countries are specialists of investment in US assets, but collateral constraints may lead to fire sale of assets. Also, CIP deviations are endogenously determined in the FX swap market. Suppliers of the FX swap are US banks faced with margin requirements while EU banks demand FX swap due to currency matching requirement.

3.1. Environment

In this model with three periods $t = 0, 1, 2$, there are two countries, the US and the EU, with the same measure of 1/2. Variables of the EU are denoted with an asterisk. Consumption goods in each country are non-perishable, implying that the storage technology of both countries has a net return rate of zero. The numeraire of the US (EU) is the US (EU) consumption good. Although there is no money in this model, the numeraire of the US and the EU are denoted as USD ($) and Euro (€) respectively for notational convenience. The spot exchange rate e and the forward exchange rate f are expressed in units of €per $, so rises in exchange rates mean appreciations of USD.

There are two types of agents in each country: depositors and bankers. The mass of depositors and bankers are both one, and they are perfectly competitive. Deposit rates of both countries are zero due to the zero net return rate of storing. We assume that agents consume only at $t = 2$ and are risk-neutral, so the utility function U_i of agent i is defined as

$$U_i = c_{i,2}$$

for consumption $c_{i,2}$ of agent $i \in \{b, d\}$.

There is only one asset in the economy: the US asset. One unit of US consumption good is converted to the asset by the production function $f(\cdot)$ where $f' > 0$, $f'' < 0$, $f(0) = 0$, and $f'(0) = \infty$. The US asset is also non-perishable as consumption goods. At $t = 2$, one unit of the asset is converted to one unit of US consumption good. This implies that the price of US asset is equal to one in the last period.

Both of US banks and EU banks can invest in US assets, meaning that they can access the conversion technology of $f(\cdot)$. On the other hand, depositors cannot use this technology, so they are not able to invest in US assets. In other words, banks are specialists in investments. As it will be specified later, banks are subject to financial frictions, so this assumption leads to fire sale of US assets during financial distress.

3.1.1. US Bank

At $t = 0$, the representative US bank is endowed with exogenous ρ_0 and issues deposits d_0 from US depositors while it invests i_0 into US assets. Hence,

$$a_0 = f(i_0) = f(\rho_0 + d_0) \tag{1}$$

where a_0 is the US asset at the end of period $t = 0$.

In period 1, the US bank receives ρ_1 exogenously, takes newly issued deposits d_1, and sells Δa amount of US assets at a price of p_1. ρ_1 is random and revealed at the beginning of $t = 1$. From these sources of funds, it repays d_0 with the deposit rate zero while it invests i_1 or transacts S_1 amount of FX swap.[3]

Figure 3 describes cash flows from FX swap contract S_1 between the US bank and the EU bank. In period 1, the US bank exchanges $\$S_1$ into €$e_1 S_1$ at the spot exchange rate e_1. At the same time, both banks enter into a forward contract at the forward exchange rate f_1 which is determined at $t = 1$. Then, at $t = 2$, the US bank returns €$e_1 S_1$ to the EU bank while getting $\$(e_1/f_1)S_1$ back. Note that $\$(e_1/f_1)S_1$ is predetermined at $t = 1$, implying that there is no exchange rate risk. The net return rate on this FX swap contract is then $e_1/f_1 - 1$. This is equivalent to the CIP deviation χ_1 since χ_1 is by definition

$$(1 + r_1^*)\frac{e_1}{f_1} - (1 + r_1)$$

for risk-free rates r_1 and r_1^* of the US and the EU respectively while deposit rates are zero.[4] This implies that FX swap trades are equivalent to arbitraging CIP deviations.

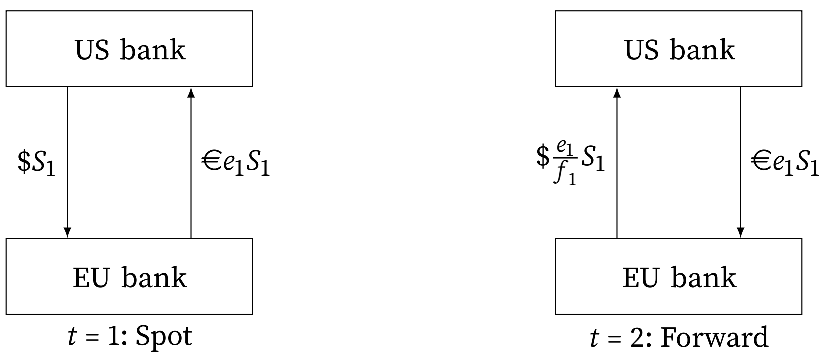

FIGURE 3. FX Swap Contract

There are two key financial frictions that the US bank faces. First, there is a limited commitment constraint on d_1.[5] When banks do not repay their deposits, depositors can seize ϕ units of US assets. I assume that depositors can seize all of FX swap S_1, implying that there is no limited commitment on S_1. This can be justified since FX swap in this paper is cash-like and thus there are relatively little transaction costs of seizing the arbitrage

[3]One may ask why the US bank does not trade FX swap at $t = 0$. It will be explained in the next section that there is no demand for FX swap by EU banks at $t = 0$ since they are assumed to be able to access direct dollar funding market at $t = 0$.

[4]The only risk-free rate in this model is the deposit rate.

[5]There is no limited commitment constraint at $t = 0$ for simplicity. As it will be specified later, this implies that the crisis happens only at the interim period $t = 1$.

capital. Then, we can obtain the following collateral constraint as

$$d_1 - S_1 \leq p_1 \phi \tag{2}$$

In addition, there is a margin requirement for FX swap (see Ivashina, Scharfstein, and Stein, 2015). When US banks arbitrage S_1, they need to set aside γ fraction of the arbitrage as a margin. In other words, the effective cost of S_1 is $1+\gamma$. Hence, the budget constraint at $t = 1$ is given by

$$i_1 + (1+\gamma)S_1 + d_0 = \rho_1 + d_1 + p_1 \Delta a \tag{3}$$

In period 2, the US bank consumes $c_{b,2}$ and repays d_1. Source of funds consist of residual assets $a_0 - \Delta a$, newly accumulated assets $f(i_1)$, and returns from CIP arbitrage $(1+\chi_1)S_1$ with margin γS_1 set aside at $t = 1$. Thus, we can obtain the following budget constraint:

$$c_{b,2} + d_1 = f(i_0) - \Delta a + f(i_1) + (1+\chi_1)S_1 + \gamma S_1 \tag{4}$$

3.1.2. EU Bank

As the US bank, the representative EU bank invests i_0^* in US assets with exogenous endowments ρ_0^* and deposits d_0^*. Then, a_0^* is determined as

$$a_0^* = f(i_0^*) = f(\rho_0^* + d_0^*) \tag{5}$$

All variables in (5) are denominated in USD. Importantly, I assume that the EU bank can issue deposits in USD at $t = 0$. This is the case when the EU bank can approach steady and ample pool of *direct* dollar funding, and they need not resort to *synthetic* dollar funding.

In the next period, the EU bank invests i_1^* and repays d_0^*. Also, it is endowed with ρ_1^*, sells Δa^* amount of assets at price p_1, and issues deposits d_1^*. In distinction from $t = 0$, it cannot issue deposits in USD directly at $t = 1$, so d_1^* is denominated in EUR. This assumption is motivated from the dry-up of direct dollar funding during financial distress periods such as European sovereign debt crisis. It became hard for European banks to borrow USD directly, so they relied on FX swap markets to fund US dollar (Ivashina, Scharfstein, and Stein, 2015). Then, the budget constraint evaluated in USD is

$$i_1^* + d_0^* = \rho_1^* + \frac{1}{e_1} d_1^* + p_1 \Delta a^* \tag{6}$$

for the spot exchange rate e_1.

As the US bank, the EU bank is also subject to a collateral constraint on deposits:

$$\frac{1}{e_1} d_1^* \leq p_1 \phi^* \tag{7}$$

where ϕ^* is the amount of US assets that EU depositors can seize.

The key assumption for the EU bank is that it should match currencies of their assets and liabilities. This assumption is realistic since banks are heavily penalized if they leave currency mismatches in their balance sheets. Since the EU bank cannot borrow USD directly at $t = 1$, the only way for currency matching is synthetic dollar funding. In other words, it has to eliminate currency mismatches by FX swap. The net need for USD is $i_1^* + d_0^* - (\rho_1^* + p_1 \Delta a^*)$, and it is met by EUR denominated deposits d_1^*. Thus, the amount of currency mismatch is exactly d_1^*/e_1, so the demand for FX swap S_1^* is

$$S_1^* = \frac{1}{e_1} d_1^* = i_1^* - (\rho_1^* - d_0^*) - p_1 \Delta a^* \tag{8}$$

Note that there is no term regarding S_1^* in period 1 budget constraint (6) since it just accompanies the exchange of currencies at $t = 1$. In other words, FX swap contracts are off-balance sheet items.[6]

In period $t = 2$, the EU bank's consumption in USD is $c_{b,2}^*$ while the repayment of deposits in USD is d_1^*/e_2. On the other hand, sources of funds in USD are remaining assets $a_0^* - \Delta a^*$, newly accumulated assets $a_1^* = f(i_1^*)$, and the net return from FX swap. Since Figure 3 shows that the EU bank exchanges $\$(e_1/f_1)S^*$ with €$e_1 S^*$, the net return in USD is $(1/e_2)d_1^* - (1/f_1)d_1^*$. Thus, the budget constraint is

$$c_{b,2}^* + \frac{1}{e_2} d_1^* = f(i_0^*) - \Delta a^* + f(i_1^*) + \frac{1}{e_2} d_1^* - \frac{1}{f_1} d_1^* \tag{9}$$

Said differently, all USD variables are evaluated by the predetermined forward exchange rate f_1.

3.1.3. Depositors

The representative US depositor is endowed with y at both of the periods $t = 0$ and $t = 1$. For the case of the EU depositor, endowments are y^* at each period. They can store or deposit endowments with the net return rate of zero. Then,

$$c_{d,2} = (y - d_0 - d_0^*) + (y + d_0 + d_0^* - d_1) + d_1 = 2y \tag{10}$$

$$c_{d,2}^* = y^* + (y^* - d_1^*) + d_1^* = 2y^* \tag{11}$$

[6] By the same reason, S_1 in the US bank's balance sheet represents holdings of EUR. The conversion of currencies is not reported on the balance sheet of the US bank.

3.1.4. Market Clearing Conditions

Since the net sales of US assets is zero in the equilibrium, the market clearing condition for the US asset market is

$$\Delta a + \Delta a^* = 0 \tag{12}$$

Meanwhile, the market clearing condition for the FX swap market is given by

$$S = S^* \tag{13}$$

3.2. First-Best Allocation

As a benchmark, I investigate the first-best allocation of this economy. The first-best allocation is defined as the solution for the social planner problem without any financial friction, *i.e.* the solution under non-binding collateral constraints and $\gamma = 0$.

DEFINITION 1. *(First-best Allocation) The first-best allocation is defined by* $\{d_0^{FB}, d_0^{*FB}, i_0^{FB}, i_0^{*FB}, d_1^{FB}, d_1^{*FB}, i_1^{FB}, i_1^{*FB}, \Delta a^{FB}, \Delta a^{*FB}, S_1^{FB}, S_1^{*FB}, c_{b,2}^{FB}, c_{b,2}^{*FB}, c_{d,2}^{FB}, c_{d,2}^{*FB}\}$ *solving the following social planner problem*

$$\max E[\frac{1}{2}(c_{b,2} + c_{d,2}) + \frac{1}{2}(c_{b,2}^* + c_{d,2}^*)]$$
$$s.t.\ (1), (3), (4), (5), (6), (8), (9), (10), (11), (12), (13) \text{ with } \gamma = 0$$

PROPOSITION 1. *The first-best allocation is characterized by* $f'(\rho_0 + d_0^{FB}) = f'\left(\rho_0^* + d_0^{*FB}\right) = 1$ *and* $f'(i_1^{FB}) = f'(i_1^{*FB}) = 1$.

PROOF. See Appendix A.1. □

The intuition behind these first-order conditions are simple “marginal benefit = marginal cost” relation. The planner decides investments at $t = 0$ and $t = 1$ until marginal returns are equal to marginal costs. The marginal cost of investment is one since the conversion technology requires one unit of US consumption good. Without the margin requirement and collateral constraints, FX swap is just transfer of wealth, not affecting marginal costs of investments by EU banks. Note that Δk^{FB} and Δk^{*FB} are indeterminate.

4. Optimal Policy

4.1. Policy Instrument

As discussed in Section 2, the swap spread ss imposes a ceiling on CIP deviations. Hence, we can consider ss as the policy instrument of central bank swap lines with an occasionally binding constraint $\chi_1 \leq ss$.

However, in general, it is more difficult to solve for an optimal price measure of a policy than an optimal quantity measure, especially when there is an occasionally binding constraint. For this reason, this paper considers the state-contingent liquidity provision through swap lines S^{SL} as the policy instrument. This is possible since there is one-to-one relationship between ss and S^{SL}. $S^{SL} \geq 0$ fills the excess demand for synthetic dollar funding when ss imposes a ceiling on CIP deviations, so S^{SL} pins down ss.

In specific, the US government borrows S^{SL} from US depositors and lends the same amount to EU banks at $t = 1$. At $t = 2$, it gets repayment of $(1 + \chi_1)S^{SL}$ and repays S^{SL} to US depositors. The remaining net return $\chi_1 S^{SL}$ is rebated to US banks.

4.2. Optimal Policy under Discretion

In this section, we investigate the optimal swap line policy under discretion. In other words, the ex-post efficient swap line policy after ρ and ρ^* are realized is derived. Thus, the discretion policy is analyzed and discussed from the point of view of period $t = 1$.

4.2.1. Optimization Problems

First, let us consider the US bank problem. The swap line policy does not change budget constraints (1), (3) and the collateral constraint (2) while period-2 budget constraint changes to

$$c_{b,2} + d_1 = f(i_0) - \Delta a + f(i_1) + (1 + \chi_1)S_1 + \gamma S_1 + \chi_1 S^{SL} \tag{14}$$

due to the rebate of $\chi_1 S^{SL}$. Combining with (1) and (3), the US bank optimization problem at $t = 1$ is given by

$$\begin{aligned} V_{b,1}(\rho_1 - d_0) \equiv \max_{i_1, \Delta a, S_1} \; & f(i_1) - i_1 - (1 - p_1)\Delta a + \chi_1 S_1 + \chi_1 S^{SL} \\ \text{s.t. } & i_1 \leq (\rho_1 - d_0) + p_1(\phi + \Delta a) - \gamma S_1 \end{aligned} \tag{15}$$

Here, $f(\rho_0 + d_0) + \rho_1 - d_0$ is omitted since it is predetermined and thus unrelated to period 1 choice. The value function $V_{b,1}(\cdot)$ of the US bank depends on the state variable $\rho_1 - d_0$, which is the available US dollar that it has at the beginning of $t = 1$.

For the Lagrangian multiplier λ_1 of the collateral constraint, the first-order conditions are derived as

$$f'(i_1) = 1 + \lambda_1 \tag{16}$$

$$p_1 = \frac{1}{f'(i_1)} \tag{17}$$

$$\chi_1 = \gamma(f'(i_1) - 1) \tag{18}$$

$$\lambda_1(\rho_1 - d_0 + p_1(\phi + \Delta a) - \gamma S_1 - i_1) = 0 \tag{19}$$

First, (16) determines the investment. If the collateral constraint does not bind and thus $\lambda_1 = 0$, then $i_1 = i_1^{FB}$. If $\lambda_1 > 0$, then $i_1 < i_1^{FB}$, *i.e.* the US bank is underinvesting at $t = 1$. Next, the price of capital is determined by the no-arbitrage condition (17) between investment and purchasing assets. When $i_1 = i_1^{FB}$, then $p_1 = 1$ while $p_1 < 1$ in the binding regime. (18) determines the CIP deviation χ_1 since one unit of FX swap requires γ units of haircut with the opportunity cost given by the net return rate on the investment $f'(i_1) - 1$.[7] (19) is the complementary slackness condition for the collateral constraint.

Next, for the case of the EU bank, budget constraints and the demand for FX swap (6), (8), (9) become

$$i_1^* + d_0^* = \rho_1^* + \frac{1}{e_1} d_1^* + p_1 \Delta a^* + S^{SL} \tag{20}$$

$$S^* = i_1^* - (\rho_1^* - d_0^*) - p_1 \Delta a^* - S^{SL} \tag{21}$$

$$c_{b,2}^* + \frac{1}{e_2} d_1^* = f(i_0^*) - \Delta a^* + f(i_1^*) + \frac{1}{e_2} d_1^* - \frac{1}{f_1} d_1^* - (1 + \chi_1) S^{SL} \tag{22}$$

Then, maximizing EU bank's consumption is

$$\begin{aligned} V_{b,1}^*(\rho_1^* - d_0^* + S^{SL}) \equiv \max_{i_1, \Delta a^*} \; & f(i_1^*) - (1 + \chi_1) i_1^* - (1 - (1 + \chi_1) p_1) \Delta a^* \\ \text{s.t.} \; & i_1^* \leq \rho_1^* - d_0^* + p_1(\phi^* + \Delta a^*) + S^{SL} \end{aligned} \tag{23}$$

ignoring $f(\rho_0^* + d_0^*) + \rho_1^* - d_0^*$. The value function of the EU bank is the function of the available US dollar $\rho_1^* - d_0^* + S^{SL}$.

The first-order conditions are

$$f'(i_1^*) = 1 + \chi_1 + \lambda_1^* \tag{24}$$

$$p_1 = \frac{1}{f'(i_1^*)} \tag{25}$$

$$\lambda_1^* (\rho_1^* - d_0^* + p_1(\phi^* + \Delta a^*) + S^{SL} - i_1^*) = 0 \tag{26}$$

for the Lagrangian multiplier λ_1^* of the collateral constraint. The marginal cost of investment by the EU bank is $1 + \chi_1 + \lambda_1^*$ where $1 + \chi_1$ is the synthetic dollar funding cost and λ^* is the shadow cost of the collateral constraint. The synthetic dollar funding cost is $1 + \chi_1$ since the currency matching assumption requires EU banks to fund USD in the FX swap market by paying CIP deviations χ_1. (25) and (26) are the same as the US bank.

Finally, consumption of US and EU depositors are given by

$$c_{d,2} = 2y - L(S^{SL}) \tag{27}$$

[7] This is essentially equivalent to Ivashina, Scharfstein, and Stein (2015) with endogenizing the alternative investment as the investment in US assets.

$$c^*_{d,2} = 2y^* \tag{28}$$

where $L(S^{SL})$ is the deadweight loss of the swap line policy. $L(S^{SL})$ is given exogenously in order to simplify the analysis.

4.2.2. Competitive Equilibrium

DEFINITION 2. *(Competitive Equilibrium) The competitive equilibrium at $t = 1$ is defined by allocations $\{d_1, d_1^*, i_1, i_1^*, \Delta a, \Delta a^*, S_1, S_1^*, c_{b,2}, c^*_{b,2}, c_{d,2} c^*_{d,2}\}$ and prices $\{p_1, \chi_1, \lambda_1, \lambda_1^*\}$ satisfying*

i. *US bank: (3), (14), (16), (17), (18),(19)*
ii. *EU bank: (20), (21), (22), (24), (25), (26)*
iii. *Depositors: (27), (28)*
iv. *Market clearing conditions: (12) and (13)*

From (17) and (25), $i_1 = i_1^*$. Investments in both countries are synchronized because asset markets are perfectly integrated. Then, $\lambda_1^* = (1-\gamma)\lambda_1$ by (16), (18), and (24). This implies that $\lambda_1 = 0$ if and only if $\lambda_1^* = 0$, *i.e.* both of the collateral constraints of the US and the EU are binding or non-binding. We can rule out the case in which one country's collateral constraint binds while the another country's collateral constraint does not bind, simplifying the analysis a lot.

First, both the US and the EU are under the normal regime when $\lambda_1 = \lambda_1^* = 0$. In this case, the competitive equilibrium attains the first-best allocation. Since $f'(i_1) = f'(i_1^*) = 1$, $i_1 = i_1^* = i_1^{FB}$. Then, $p_1 = 1$, so the asset price stays constant over time. Also, $\chi_1 = 0$, *i.e.* covered interest rate parity holds. Here, Δa and Δa^* are indeterminate since sales of assets yield zero profit for banks. Other variables are obtained by corresponding budget constraints.

The normal regime is realized when collateral constraints do not bind at the first-best allocation:

$$\begin{aligned} i_1^{FB} &< \rho_1 - d_0 + \phi - \gamma(i_1^{FB} - (\rho_1^* - d_0^*) - S^{SL}) \\ i_1^{FB} &< \rho_1^* - d_0^* + \phi^* + S^{SL} \end{aligned}$$

We can rearrange these inequalities as

$$(\rho_1 - d_0) + \gamma(\rho_1 - d_0^* + S^{SL}) > (1+\gamma)i_1^{FB} - \phi \tag{29}$$

$$\rho_1^* - d_0^* + S^{SL} > i_1^{FB} - \phi^* \tag{30}$$

Here, $\rho_1 - d_0$ and $\rho_1^* - d_0^* + S^{SL}$ are the US dollar available to US banks and EU banks respectively. The inequalities mean that both banks need sufficient amount of USD for their collateral constraints to be non-binding.

Figure 4 displays the conditions for the normal regime. If $\rho_1 - d_0$ and $\rho_1^* - d_0^* + S^{SL}$ are inside the shaded area, the normal regime is realized. Otherwise, λ_1 and λ_1^* are strictly positive, which is defined as the crisis regime. As ρ_1^* declines, ρ_1 needs to be higher for the normal regime. Also, if $\rho_1 - d_0^* + S^{SL}$ is below a threshold $i_1^{FB} - \phi^*$, the crisis regime is always realized.

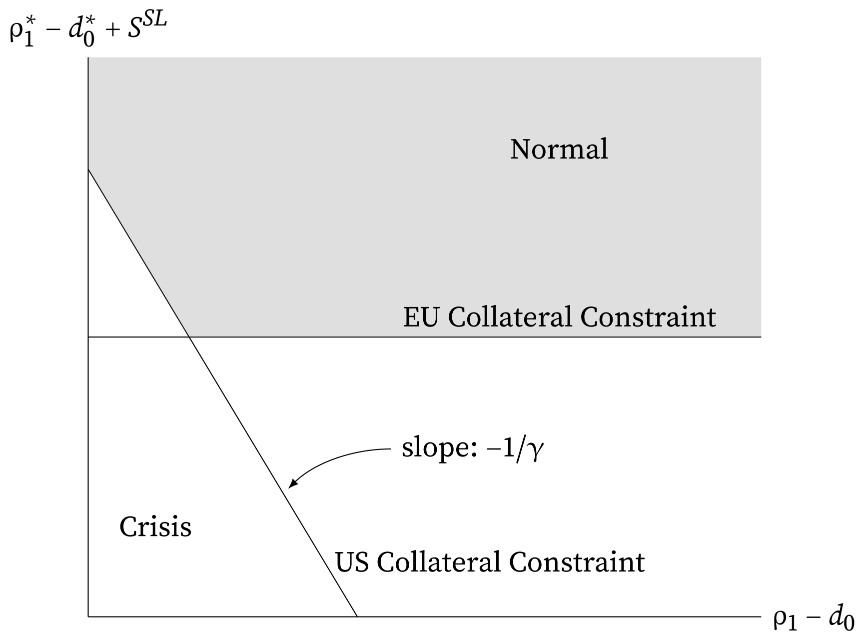

FIGURE 4. Normal and Crisis Regimes

Next, let us consider the crisis regime when $\lambda_1 = \lambda_1^* > 0$. Then, $p_1 < 1$ and $\chi_1 > 0$. The EU bank's collateral constraint implies

$$\Delta a^* = f'(i_1)\left(i_1 - (\rho_1^* - d_0^* + S^{SL}) - \frac{1}{f'(i_1)}\phi^*\right)$$

Plugging this into the US bank's collateral constraint,

$$i_1 = \frac{1}{2}\left[(\rho_1 - d_0) + (\rho_1^* - d_0^* + S^{SL}) + \frac{1}{f'(i_1)}\left(\phi + (1-\gamma)\phi^*\right)\right] \equiv G(i_1; m_1)$$

where $m_1 \equiv (\rho_1 - d_0) + (\rho_1^* - d_0^* + S^{SL})$ is the total available US dollar at the beginning of $t = 1$. Thus, i_1 is the fixed point of $G(i_1; m_1)$.

LEMMA 1. *i_1 of the crisis regime uniquely exists if for all $i_1 < i_1^{FB}$ there exists $c < 1$ such that*

$$G'(i_1) = -\frac{f''(i_1)}{f'(i_1)^2}\frac{1}{2}\left(\phi + (1-\gamma)\phi^*\right) \leq c$$

PROOF. Banach fixed point theorem. □

From now on, we assume that the above condition holds so that we can rule out multiple fixed points. Then, i_1 is uniquely determined, and we can obtain all other variables as well. Note that $-f''(i_1)/f'(i_1)^2$ is equal to $\partial p_1/\partial i_1$ since $p_1 = 1/f'(i_1)$ in the equilibrium. As it will be discussed soon, this term is related to a pecuniary externality. Since $f'' < 0$, $G'(i_1) > 0$ while $G'(i_1) = 0$ if there is no pecuniary externality.

Before moving on to the Ramsey problem, we look at effects of the swap line policy on economic variables at $t = 1$. First, let us begin with the investment i_1 because all other variables are determined as functions of i_1. If i_1^{FB} is attained, then obviously $\partial i_1/\partial S^{SL} = 0$. From now on, we will focus on the crisis regime.

PROPOSITION 2. *(Effects of Swap Lines on Investments) In the crisis regime,*

$$\frac{\partial i_1}{\partial S^{SL}} = \frac{\partial i_1^*}{\partial S^{SL}} = \frac{1}{2}\frac{1}{1 - G'(i_1)}$$

Also, $\frac{\partial i_1}{\partial S^{SL}}$ is strictly positive if and only if i_1 uniquely exists.

PROOF. See Appendix A.2. □

Proposition 2 implies the existence of the financial accelerator due to the pecuniary externality. $G'(i_1)$ is the term showing the pecuniary externality. When banks choose the level of i_1, they do not internalize the effect of i_1 on $p_1 = 1/f'(i_1)$, which is $-f''(i)/f'(i)^2$. This pecuniary externality has real effects since there are collateral constraints affected by p_1. As i_1 increases, $p_1\phi$ and $p_1\phi^*$ go up, relaxing US and EU collateral constraints. On the other hand, as i increases, $S^* = p_1\phi^*$ also increases, tightening the US collateral constraint by a factor of γ due to the margin requirement. Thus, $G'(i_1)$ measures the pecuniary externality. Then, there is a feedback between i_1 and $G'(i_1)$, yielding the amplification effect as Proposition 2. Since $G'(i_1) > 0$ when there is pecuniary externality, $\partial i_1/\partial S^{SL} > 1/2$. Without the pecuniary externality, $\partial i_1/\partial S^{SL} = 1/2$.

Then, an increase in S^{SL} leads to higher p_1 and lower χ_1:

$$\frac{\partial p_1}{\partial S^{SL}} = -\frac{f''(i_1)}{f'(i_1)^2}\frac{1}{2}\frac{1}{1 - G'(i_1)} > 0$$
$$\frac{\partial \chi}{\partial S^{SL}} = \gamma f''(i_1)\frac{1}{2}\frac{1}{1 - G'(i_1)} < 0$$

Liquidity provision through swap lines mitigates the fall in asset price and frictions in the FX swap market since the additional liquidity relaxes collateral constraints.

Finally, I investigate how the private FX swap market reacts to the central bank swap lines. Since $S_1 = S_1^* = p_1\phi^*$ in the crisis regime,

$$\frac{\partial S_1}{\partial S^{SL}} = -\phi^* \frac{f''(i_1)}{f'(i_1)^2}\frac{1}{2}\frac{1}{1-G'(i_1)} > 0$$

This implies that there is no crowding-out effect of the swap line policy by central banks. The total synthetic dollar funding $S_1 + S^{SL}$ rises more than one-to-one with S^{SL}.

4.2.3. Ramsey Problem

The world welfare function is given by

$$\begin{aligned}&\frac{1}{2}(c_{b,2} + c_{d,2}) + \frac{1}{2}(c^*_{b,2} + c^*_{d,2})\\ &\quad = f(\rho_0 + d_0) + \rho_1 - d_0 + f(i_1) - i_1 + 2y - L(S^{SL}) + f(\rho_0^* + d_0^*) + \rho_1^* - d_0^* + f(i_1^*) - i_1^* + 2y^*\end{aligned}$$

When we solve $t = 1$ problem, we can ignore terms like $f(\rho_0 + d_0) + \rho_1 - d_0$ and $f(\rho_0^* + d_0^*) + \rho_1^* - d_0^*$ which are predetermined at $t = 1$.

Under the optimized choices of bankers, i_1 is a function of the total amount of USD m_1. Then, at $t = 1$, a global Ramsey planner chooses state-contingent S^{SL} to solve the following problem

$$W_1(m_1) \equiv \max_{S^{SL}} \frac{1}{2}\left[f(i_1(m)) - i_1(m) - L(S^{SL}) + 2y\right] + \frac{1}{2}\left[f(i_1^*(m)) - i_1^*(m) + 2y^*\right] \tag{31}$$

Here, we are implicitly assuming that one Ramsey planner maximizes the world welfare. One may argue that this is an unrealistic description of the real world; in practice, there are several Ramsey planners who care only about their own jurisdiction. In Section 4.6, I introduce a cooperative Ramsey problem where Ramsey planners of the US and the EU coordinate the swap line policy S^{SL} with certain bargaining power. It is shown that the global Ramsey problem is the special case of the cooperative Ramsey problem where the bargaining power is equal to the relative size of the economy, 1/2 in this model.

If the economy is at the normal regime, then $i_1 = i_1^{FB}$ regardless of S^{SL}. In this case, the following Ramsey problem

$$\max_{S^{SL}} \frac{1}{2}\left[f(i_1^{FB}) - i_1^{FB} - L(S^{SL}) + 2y\right] + \frac{1}{2}\left[f(i_1^{FB}) - i_1^{FB} + 2y^*\right]$$

yields $S^{SL} = 0$ since S^{SL} introduces only deadweight loss.

On the other hand, if the economy is at the crisis regime, the first-order condition of (31) is

$$(f'(i_1) - 1)\frac{\partial i_1}{\partial S^{SL}} + (f'(i_1^*) - 1)\frac{\partial i_1^*}{\partial S^{SL}} = \frac{\lambda_1}{1 - G'(i_1)} = L'(S^{SL}) \tag{32}$$

since $f'(i_1) - 1 = \lambda_1$ and $f'(i_1^*) - 1 = \chi_1 + \lambda_1^* = \lambda_1$. The LHS of (32) is the marginal benefit of S^{SL} while the RHS is the marginal cost. One unit increase in S^{SL} increases investments in both countries by $\partial i_1 / \partial S^{SL}$ respectively, which has marginal value of $f'(i_1) - 1$. Note that the marginal value of i_1^* is also $f'(i_1^*) - 1$, rather than $f'(i_1^*) - (1 + \chi_1)$, since χ_1 is transfer of wealth from the EU to the US and thus cancelled from the point of view of world economy.[8] We can also consider the marginal benefit as the financial accelerator effect on relaxing collateral constraints. S^{SL} relaxes collateral constraints, yielding shadow value of λ_1, and it is amplified at the rate of the pecuniary externality $G'(i_1)$.

Since $\lambda_1 > 0$ in the crisis regime, $L' > 0$ and $L'(0) = 0$, $S^{SL} > 0$. In other words, the US government provides non-zero liquidity to EU banks whenever there is a crisis. On the other hand, since $L' > 0$, λ_1 is strictly positive. This implies that the economy is still at the crisis regime in spite of swap lines due to the deadweight loss. Proposition 3 summarizes these arguments.

PROPOSITION 3. *(Optimal Discretion Policy for Central Bank Swap Lines) After normal or crisis regimes are realized, the optimal central bank swap line policy S^{SL} is given by the following:*

i. $S^{SL} = 0$ *if the economy is under the normal regime.*
ii. S^{SL} *is determined by (32) if the economy is under the crisis regime.*
iii. *Under the crisis regime, $S^{SL} > 0$ while S^{SL} is partial in the sense that it does not yield a full recover from the crisis regime.*

4.3. Overborrowing

We analyze how the ex-post efficient policy may lead to an overborrowing and higher probability of financial crises from the ex-ante point of view in the absence of macroprudential policies.

In period 0, the representative US bank solves

$$\max_{d_0} E\left[f(\rho_0 - d_0) + \rho_1 - d_0 + V_{b,1}(\rho_1 - d_0)\right]$$

where $V_{b,1}(\rho_1 - d_0)$ is the value function of period-1 optimization problem (15). Since $V'_{b,1}(\rho_1 - d_0) = \lambda_1$ by envelope condition, the first-order condition for the period-0 problem is given by

$$f'(\rho_0 - d_0) = 1 + E[\lambda_1] \tag{33}$$

The LHS is the marginal benefit of one unit of deposits d_0. The marginal cost in the RHS consists of the physical cost of deposits and the expected shadow cost. When d_0 increases,

[8] This is not the case when we consider the policy coordination problem and the bargaining power is different from the relative size of the economy. In that case, the transfer of wealth is not cancelled and is valued larger or less depending on the bargaining power of the US.

then the US collateral constraint becomes tightened with the expected shadow cost of $E[\lambda_1]$.

Similarly, in period 0, the EU bank solves

$$\max_{d_0^*} E\Big[f(\rho_0^* + d_0^*) + (1+\chi_1)(\rho_1^* - d_0^*) + V_{b,1}^*(\rho_1^* - d_0^*)\Big]$$

for the value function $V_{b,1}^*(\cdot)$ in (23), which yields the following first-order condition as

$$f'(\rho_0^* + d_0^*) = 1 + E[\chi_1 + \lambda_1^*] = 1 + E[\lambda_1] \tag{34}$$

Next, choices of d_0 and d_0^* by US banks and EU banks are compared with the ones of the social planner.[9] For the value function $W_1(m_1)$ of period-1 Ramsey problem, the social planner maximizes the world welfare by choosing d_0 and d_0^*:

$$\max_{d_0, d_0^*} E\Big[\frac{1}{2}\{f(\rho_0 + d_0) + \rho_1 - d_0\} + \frac{1}{2}\{f(\rho_0^* + d_0^*) + \rho_1^* - d_0^*\} + W_1(m_1)\Big] \tag{35}$$

Here, d_0 and d_0^* solving this problem are constrained efficient amount of period-0 borrowing given period-1 optimal swap line policy. By the envelope condition,

$$W_1'(m_1) = \frac{1}{2}(f'(i_1) - 1)\frac{\partial i_1}{\partial m_1} + \frac{1}{2}(f'(i_1^*) - 1)\frac{\partial i_1^*}{\partial m_1} = \frac{1}{2}\frac{\lambda_1}{1 - G'(i_1)}$$

Then, the first-order conditions are

$$f'(\rho_0 + d_0) = 1 + E\left[\frac{\lambda_1}{1 - G'(i_1)}\right] \tag{36}$$

$$f'(\rho_0^* + d_0^*) = 1 + E\left[\frac{\chi_1 + \lambda_1^*}{1 - G'(i_1)}\right] = 1 + E\left[\frac{\lambda_1}{1 - G'(i_1)}\right] \tag{37}$$

Comparing (36) and (37) with (33) and (34), constrained efficient d_0 and d_0^* are lower than optimal choices by US and EU banks since $G'(i_1) > 0$. The main cause of this overborrowing problem is the pecuniary externality. When d_0 or d_0^* increases and collateral constraints are binding, then i_1 should decrease or part of assets should be sold. In any case, p_1 declines, tightening collateral constraints further. The social planner takes this into account, equating the marginal benefit of d_0 and d_0^* with the social marginal cost which is higher than the private marginal cost by the amplification factor of $1/(1 - G'(i_1))$. Hence, private choices of d_0 and d_0^* are higher than constrained efficient d_0 and d_0^*.

[9] The social planner problem here is different from period-0 Ramsey problem because implementability conditions (33) and (34) are ignored. The social planner problem is for the purpose of comparing between banks' choices and constrained efficient choices and showing overborrowing.

4.4. Optimal Policy under Commitment

Suppose that the US government can commit state-contingent S^{SL} at period 0, *before* ρ_1 and ρ_1^* are determined. Even though the economy is under the crisis regime at $t = 1$ depending on realizations of ρ_1 and ρ_1^*, the government does not change S^{SL}. In that sense, S^{SL} ex-ante efficient.

The Ramsey problem is defined as maximizing the expected world welfare subject to implementability conditions. Here, implementability conditions are investment function and optimality conditions (33) and (34) for d_0 and d_0^*. To summarize, the Ramsey problem is given by

$$\begin{aligned}\max_{d_0,d_0^*,S^{SL}} E\Big[&\frac{1}{2}\{f(\rho_0+d_0)+\rho_1-d_0+f(i_1(m))-i_1(m)+2y-L(S^{SL})\}\\ &+\frac{1}{2}\{f(\rho_0^*+d_0^*)+\rho_1^*-d_0^*+f(i_1^*(m))-i_1^*(m)+2y^*\}\Big] \quad (38)\\ \text{s.t. } & f'(\rho_0+d_0)=1+E[\lambda_1]\\ & f'(\rho_0^*+d_0^*)=1+E[\lambda_1]\end{aligned}$$

Let us denote Lagrangian multipliers of implementability conditions as ν_0 and ν_0^* respectively.

PROPOSITION 4. *(Optimal Commitment Policy for Central Bank Swap Lines)*

i. $S^{SL} = 0$ *if the economy is under the normal regime.*

ii. S^{SL} *is determined by*

$$\frac{\lambda}{1-G'(i_1)}+(\nu_0+\nu_0^*)\frac{f''(i_1)}{1-G'(i_1)}=L'(S^{SL}) \quad (39)$$

if the economy is under the crisis regime.

iii. S^{SL} *under commitment is lower than* S^{SL} *under discretion in the crisis regime.*

PROOF. See Appendix A.3. □

The reason why S^{SL} under commitment is lower than S^{SL} under discretion is because $\nu_0+\nu_0^* > 0$, *i.e.* at least one of (33) and (34) is binding. Suppose that $\nu_0 = \nu_0^* = 0$ and thus both of implementability conditions are redundant. Ignoring implementability conditions, the Ramsey problem (38) becomes equivalent to (35). Then, d and d^* should satisfy constraind efficiency conditions (36) and (37). However, since $G'(i_1) > 0$, implementability conditions (33) and (34) are violated, which is contradiction.

Intuitively speaking, the Ramsey planner takes the effect of S^{SL} on ex-ante borrowing since banks tend to overborrow at $t = 0$. The overborrowing behavior is captured by the *change* in λ_1. As the US government provides S^{SL}, λ declines as

$$\frac{\partial \lambda_1}{\partial S^{SL}} = \frac{\partial \lambda_1}{\partial i_1}\frac{\partial i_1}{\partial S^{SL}} = \frac{f''(i_1)}{1-G'(i_1)} < 0$$

Since d_0 and d_0^* need to satisfy $f'(\rho_0 + d_0) = f'(\rho_0^* + d_0^*) = 1 + E[\lambda_1]$, decreasing λ_1 means higher d_0 and d_0^*. However, this makes d_0 and d_0^* far more different from their constrained efficient levels since it becomes more likely for the crisis regime to happen. The value of this overborrowing is captured by the total shadow cost $\nu_0 + \nu_0^*$, which is strictly positive. Hence, the marginal benefit of swap lines decrease by the amount of *overborrowing cost* captured by $(\nu_0 + \nu_0^*)f''(i_1)/(1 - G'(i_1))$.

4.5. Policy Mix with Macruprudential Policy

The reason why the economy does not attain constrained efficiency is because there are not enough policy instruments. In this section, we seek an additional policy tool to mitigate (or eliminate) the overborrowing problem: macroprudential policy.

Suppose that both the US and the EU governments impose taxes on period-0 deposits. Let us denote tax rates as $\tau_{d,0}$ and $\tau_{d,0}^*$ respectively. Also, it is assumed that taxes collected are rebated to banks in each jurisdiction. Then, period-0 problems of the US bank and the EU bank are given by

$$\max_{d_0} E\Big[f(\rho + (1-\tau_{d,0})d_0 + \tau_{d,0}\tilde{d}_0) + \rho_1 - d_0 + V_{b,1}(\rho_1 - d_0)\Big]$$
$$\max_{d_0^*} E\Big[f(\rho_0^* + (1-\tau_{d,0}^*)d_0^* + \tau_{d,0}^*\tilde{d}_0^*) + (1+\chi_1)(\rho_1^* - d_0^*) + V_{b,1}^*(\rho_1^* - d_0^*)\Big]$$

Here, $\tilde{d}_0$ and $\tilde{d}_0^*$ denote aggregate deposits, rather than deposits of individual representative banks. This implies that banks do not internalize tax rebates. Then, first-order conditions are

$$(1-\tau_{d,0})f'(\rho + d_0) = 1 + E[\lambda_1] \tag{40}$$
$$(1-\tau_{d,0}^*)f'(\rho_0^* + d_0^*) = 1 + E[\lambda_1] \tag{41}$$

Note that $d_0 = \tilde{d}_0$ and $d_0^* = \tilde{d}_0^*$ in the equilibrium. Compared to (33) and (34) without macroprudential policy, marginal benefits of period-0 deposits become lower due to taxes.

PROPOSITION 5. *(Optimal Tax Rate) Optimal tax rates τ and τ^* achieving constrained efficient d and d^* are given by*

$$\tau_{d,0} = \tau_{d,0}^* = \frac{E\left[\left(\frac{1}{1-G'(i_1)} - 1\right)\lambda_1\right]}{f'(\rho_0 + d_0)} \tag{42}$$

PROOF. See Appendix A.4. □

The optimal tax rate is proportional to the difference between the social marginal cost and the private marginal cost of deposits. The difference is equal to the net amplification due to pecuniary externality. This optimal tax rate corrects the overborrowing problem, achieving constrained efficiency.

COROLLARY 1. *(Policy Mix) S^{SL} under commitment combined with the optimal tax rate (42) is equivalent to S^{SL} under discretion.*

PROOF. See Appendix A.5. □

Intuitively, since optimal tax rates correct overborrowing problem, the Ramsey planner does not have to worry about the effect of S^{SL} on ex-ante borrowing. When crisis happens, the US government provides larger US dollar liquidity to foreign countries, as long as the marginal benefit is higher than deadweight loss. The potential threat to the ex-ante financial stability is dealt with macroprudential policies such as taxes on deposits.

4.6. Policy Coordination

Until now, the optimal policy has been derived from a global Ramsey planner problem maximizing the world welfare function. However, in practice, it may be unrealistic that the swap line policy is determined to maximize the world welfare. Instead, it may be decided solely by the Federal Reserve which is interested in only the US welfare. More realistically, two planners focusing only on their own countries sit on a table together and coordinate the policy.

From the above motivation, I assume that the US and EU planner jointly decide S^{SL} to maximize an weighted average of welfare of two countries (see Benigno and Benigno, 2003, Faia and Monacelli, 2004; Corsetti and Pesenti, 2005; Chari, Nicolini, and Teles, 2023). Here, the weight on each country's welfare is the bargaining power of that country. The *cooperative* Ramsey problem at $t = 1$ is then defined as

$$\max_{S^{SL}} \alpha\big[f(i_1(m_1)) - i_1(m_1) + 2y - L(S^{SL}) + TW_1\big] + (1-\alpha)\big[f(i_1^*(m)) - i_1^*(m) + 2y^* - TW_1\big]$$

where $TW_1 \equiv \chi_1(m_1)(S_1(m_1) + S^{SL}) + (1 - p_1(m_1))\Delta a^*(m_1)$ is transfer of wealth from the EU to the US. The baseline analysis is the special case of this cooperative Ramsey problem where $\alpha = 1/2$, which is the physical measure of the US and the EU.[10] In that case, TW_1 is cancelled by market clearing conditions. $\alpha = 1$ is the extreme case when the US (source country) determines the swap line policy only for the US economy. When $\alpha = 0$, then the EU (recipient country) decides the policy.

[10] 1/2 is not a special number, but it is just an example of the physical measure of each country. Thus, generically, the global Ramsey problem is the special case of the cooperative Ramsey problem when the bargaining power of the US (EU) is equal to the relative physical measure of the US (EU).

The optimality condition of the cooperative Ramsey problem is given by the following equation:

$$\frac{1}{2}\left[\frac{\lambda_1}{1-G'(i_1)}-L'(S^{SL})\right]+(2\alpha-1)\left[\frac{\partial TW_1}{\partial m_1}-\frac{1}{2}L'(S^{SL})\right]=0 \tag{43}$$

In the LHS, the first term is equal to zero under the global Ramsey problem. However, there are additional terms in this condition. First, in response to swap lines, transfer of wealth TW_1 changes by $\partial TW_1/\partial m_1$. If $\alpha > (<)1/2$, then transfer to the US TW_1 should be given positive (negative) weight $2\alpha-1$. Therefore, $(2\alpha-1)*\partial TW_1/\partial m_1$ is the additional marginal benefit of swap lines. On the other hand, $\alpha > 1/2$ means higher weight on deadweight loss $L(S^{SL})$ because $L(S^{SL})$ is born sole by the US. Hence, $(2\alpha-1)*(L'(S^{SL})/2)$ is the additional marginal cost of swap lines.

LEMMA 2. *If* $(\rho_1-d_0)+(\phi-\gamma\phi^*)/f'(i_1) > (\rho_1^*-d_0^*)+\phi^*/f'(i_1)$, *then* $\partial TW_1/\partial m_1 < 0$.

PROOF. See Appendix A.6. □

This condition means that the US bank has more available US dollar than the EU bank at $t=1$. (ρ_1-d_0) is the US dollar that the US bank has at the beginning of $t=1$. In addition to this, the US bank can borrow $\phi/f'(i_1)$ amount of USD. However, it has to set aside $\gamma\phi^*/f'(i_1)$ of as an haircut, which cannot be used. $(\rho_1^*-d_0^*)+\phi^*/f'(i_1)$ is the available US dollar that the EU bank has at the end of $t=1$.

Under the above condition, the term in the second bracket of (43) is always negative. Then, we can get this following relationship between solutions under the global Ramsey problem and the cooperative Ramsey problem.

PROPOSITION 6. *(Optimal Policy under Cooperative Ramsey Problem)*

i. *If* $\alpha > 1/2$, *then* S^{SL} *is lower than the global Ramsey solution, i.e.* S^{SL} *is undersupplied.*
ii. *If* $\alpha < 1/2$, *then* S^{SL} *is larger than the global Ramsey solution, i.e.* S^{SL} *is oversupplied.*

For an intuition, let us consider the case $\alpha = 1$, *i.e.* the US planner determines the policy. Then, she does not take the spillover to the EU into account; she only cares about the spillback effect. This means that the marginal deadweight cost of S^{SL} is larger than the marginal benefit at the solution of the global Ramsey problem. Moreover, S^{SL} decreases transfer of wealth to the US since the CIP deviation χ_1 becomes lower. Even though the quantity of synthetic dollar funding increases, the total amount of transfer of wealth declines. This makes the marginal benefit of S^{SL} even lower. Therefore, S^{SL} is undersupplied when it is determined only by the US.

5. Conclusion

This paper studies the optimal central bank swap line policy based on a two-country (US and EU) three-period model. Both US banks and EU banks invest in US assets by issuing deposits. The distinguishing part of this model is the FX swap market. EU banks demand FX swap since there is a currency mismatch between their deposits denominated in Euro and their assets denominated in US dollar. CIP deviations are costs of this currency hedging in the FX swap market. On the other hand, US banks supply FX swap and obtain CIP deviations as profits, which arises due to a margin requirement for the FX swap trade.

Both banks are subject to collateral constraints on their deposits, and these constraints depend on the US asset price. When the constraints do not bind, then the economy can attain the first-best allocation. In contrast, fire sales of assets happens when the constraints are binding, leading to a plunge in the asset price and lower investment. CIP deviations become larger since banks' intermediation activities for FX swap interrupted. Moreover, all these effects are amplified due to a pecuniary externality in collateral values.

Given that crises happen, providing dollar liquidity to EU banks through central bank swap lines can mitigate fire sales and stabilize markets. In particular, since EU banks have more dollar liquidity, their demand for synthetic dollar funding declines, leading to lower CIP deviations. Then, their investment increases and fire sales decrease, which spills back to the US by stabilizing the US asset price. From the world welfare point of view, the ex-post efficiency condition for swap lines is the equality between the marginal benefit for the world economy and the marginal deadweight loss of funding the dollar liquidity.

However, expecting the liquidity provision through swap lines, banks have incentive to increase their ex-ante borrowing from depositors. When ex-ante deposits become larger, then the probability of crises becomes higher. This larger ex-ante borrowing is not constrained efficient since banks do not take the effect of their actions on asset prices into account. Due to this pecuniary externality, there is an overborrowing problem. The ex-ante efficient policy considers the marginal cost from this overborrowing problem and provides less dollar liquidity through swap lines. In other words, there is a time inconsistency problem.

The time inconsistency problem can be resolved by introducing taxes on ex-ante borrowing. By imposing taxes on ex-ante borrowing, we can resolve overborrowing and achieve constrained efficiency. Under the optimal policy mix with macroprudential policy, the ex-ante efficient policy becomes equivalent to the ex-post efficient policy.

Finally, I discuss policy coordination between planners in each country. When planners in the US and the EU coordinate the swap line policy with a certain bargaining power, then the optimal policy is different from the one derived from the global welfare maximization problem. Under the condition that US banks have more dollar liquidity than EU banks, dollar liquidity provided through swap lines is lower (larger) than the baseline case if the US has higher (lower) bargaining power.

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Appendix A. Proofs

A.1. Proposition 1

From (1), (3), (4),

$$c_{b,2} = f(\rho_0 + d_0) + \rho_1 - d_0 + f(i_1) - i_1 - (1 - p_1)\Delta a + \chi_1 S_1$$

Also, (5), (6), (9) suggest

$$c^*_{b,2} = f(\rho^*_0 + d^*_0) + (1 + \chi_1)(\rho^*_1 - d^*_0) + f(i^*_1) - (1 + \chi_1)i^*_1 - (1 - (1 + \chi_1)p_1)\Delta a^*$$

By the market clearing conditions (12) and (13), the social planner problem becomes

$$\begin{aligned}
&\max_{i_1, i^*_1, d_0, d^*_0} E\left[\frac{1}{2}(c_{b,2} + c_{d,2}) + \frac{1}{2}(c^*_{b,2} + c^*_{d,2})\right] \\
&= \frac{1}{2}E\left[f(\rho_0 + d_0) + \rho_1 - d_0 + f(i_1) - i_1 + 2y + f(\rho^*_0 + d^*_0) + \rho^*_1 - d^*_0 + f(i^*_1) - i^*_1 + 2y^*\right]
\end{aligned}$$

The first-order conditions of the social planner problem are

$$\begin{aligned}
f'(i_1^{FB}) &= f'(i_1^{*FB}) = 1 \\
f'(\rho_0 + d_0^{FB}) &= f'(\rho^*_0 + d_0^{*FB}) = 1
\end{aligned}$$

A.2. Proposition 2

By total differentiating $i_1 = G(i_1; m_1)$,

$$\frac{\partial i_1}{\partial S^{SL}} = \frac{\partial G}{\partial i_1}\frac{\partial i_1}{\partial S^{SL}} + \frac{\partial G}{\partial m}\frac{\partial m}{\partial S^{SL}}$$

As $m_1 = (\rho_1 - d_0) + (\rho^*_1 - d^*_0 + S^{SL})$,

$$\frac{\partial i_1}{\partial S^{SL}} = G'(i_1)\frac{\partial i_1}{\partial S^{SL}} + \frac{1}{2}$$

Thus,

$$\frac{\partial i_1}{\partial S^{SL}} = \frac{1}{2}\frac{1}{1 - G'(i_1)}$$

A.3. Proposition 4

Under the normal regime, $i_1 = i_1^{FB}$ and $\lambda_1 = 0$. Hence, as the discretion policy, S^{SL} introduces only deadweight loss. This implies that $S^{SL} = 0$ under the normal regime.

Let us consider the crisis regime. First-order conditions for the Ramsey problem (38) are obtained as

$$E\left[\frac{1}{2}\Big(f'(\rho_0+d_0)-1-\frac{\lambda_1}{1-G'(i_1)}\Big)-\nu_0\Big(\frac{1}{2}\frac{f''(i_1)}{1-G'(i_1)}+f''(\rho_0+d_0)\Big)-\nu_0^*\frac{1}{2}\frac{f''(i_1)}{1-G'(i_1)}\right]=0$$
$$E\left[\frac{1}{2}\Big(f'(\rho_0^*+d_0^*)-1-\frac{\lambda_1}{1-G'(i_1)}\Big)-\nu_0\frac{1}{2}\frac{f''(i_1)}{1-G'(i_1)}-\nu_0^*\Big(\frac{1}{2}\frac{f''(i_1)}{1-G'(i_1)}+f''(\rho_0^*+d_0^*)\Big)\right]=0$$
$$\frac{\lambda_1}{1-G'(i_1)}+(\nu_0+\nu_0^*)\frac{f''(i_1)}{1-G'(i_1)}=L'(S^{SL})$$

Combining the first two equations, since $\rho_0+d_0=\rho_0^*+d_0^*$,

$$\nu_0+\nu_0^*=\frac{E\left[\Big(1-\frac{1}{1-G'(i_1)}\Big)\lambda_1\right]}{E\left[\frac{f''(i_1)}{1-G'(i_1)}\right]+f''(\rho_0+d_0)}$$

Since $f''<0$ and $G'(i_1)>0$, $\nu_0+\nu_0^*>0$. Then,

$$\frac{\lambda_1}{1-G'(i_1)}+(\nu_0+\nu_0^*)\frac{f''(i_1)}{1-G'(i_1)}<\frac{\lambda_1}{1-G'(i_1)}$$

where the RHS of the inequality is the marginal benefit of swap lines under discretion. Since L' is an increasing function, S^{SL} under commitment is lower than under discretion.

A.4. Proposition 5

Optimal choices for d_0 and d_0^* by US banks and EU banks satisfy

$$f'(\rho_0+d_0)=\frac{1+E[\lambda_1]}{1-\tau_{d,0}}$$
$$f'(\rho_0^*+d_0^*)=\frac{1+E[\lambda_1]}{1-\tau_{d,0}^*}$$

while the constrained efficiency conditions are

$$f'(\rho_0+d_0)=1+E\left[\frac{\lambda_1}{1-G'(i_1)}\right]$$
$$f'(\rho_0^*+d_0^*)=1+E\left[\frac{\lambda_1}{1-G'(i_1)}\right]$$

Then, constrained efficient d_0 and d_0^* are attained if and only if

$$\frac{1+E[\lambda_1]}{1-\tau_{d,0}}=1+E\left[\frac{\lambda_1}{1-G'(i_1)}\right]$$

$$\frac{1+E[\lambda_1]}{1-\tau_{d,0}^*} = 1+E\left[\frac{\lambda_1}{1-G'(i_1)}\right]$$

Rearranging these two equations, we can get optimal tax rates as

$$\tau_{d,0} = \tau_{d,0}^* = \frac{E\left[\left(\frac{1}{1-G'(i_1)}-1\right)\lambda_1\right]}{f'(\rho_0+d_0)}$$

A.5. Corollary 1

Given optimal tax rates $\tau_{d,0}$ and $\tau_{d,0}^*$, the Ramsey problem at $t=0$ is

$$\begin{aligned}
\max_{d_0,d_0^*,S^{SL}} E\Big[&\frac{1}{2}\{f(\rho_0+d_0)+\rho_1-d_0+f(i_1(m_1))-i_1(m_1)+2y-L(S^{SL})\} \\
&+\frac{1}{2}\{f(\rho_0^*+d_0^*)+\rho_1^*-d_0^*+f(i_1^*(m_1))-i_1^*(m_1)+2y^*\}\Big] \\
\text{s.t.}\ &(1-\tau_{d,0})f'(\rho_0+d_0)=1+E[\lambda_1] \\
&(1-\tau_{d,0}^*)f'(\rho_0^*+d_0^*)=1+E[\lambda_1]
\end{aligned}$$

Note that there are not $\tau_{d,0}$ and $\tau_{d,0}^*$ in the objective function due to rebates of taxes. Let us ignore implementability conditions. Then, first-order conditions are

$$\begin{aligned}
f'(\rho_0+d_0) &= 1+E\left[\frac{\lambda_1}{1-G'(i_1)}\right] \\
f'(\rho_0^*+d_0^*) &= 1+E\left[\frac{\lambda_1}{1-G'(i_1)}\right] \\
\frac{\lambda_1}{1-G'(i_1)} &= L'(S^{SL})
\end{aligned}$$

First two equations are equivalent to implementability conditions under optimal tax rates. This implies that implementability conditions are indeed redundant. The last equation is equivalent to the optimality condition for the discretion policy (32).

A.6. Lemma 2

Differentiation TW_1 with respect to S^{SL},

$$\begin{aligned}
\frac{\partial TW_1}{\partial S^{SL}} = &-(1-\gamma)(f'(i_1)-1)+\gamma f''(i_1)\left(\frac{\phi^*}{f'(i_1)^2}+S^{SL}\right)\frac{\partial i_1}{\partial m_1} \\
&+f''(i_1)\frac{\partial i_1}{\partial m_1}(i_1-(\rho_1^*-d_0^*)-\frac{\phi^*}{f'(i_1)^2}-S^{SL})
\end{aligned}$$

$$< -(1-\gamma)(f'(i_1)-1) + \gamma f''(i_1)\Big(\frac{\phi^*}{f'(i_1)^2} + S^{SL}\Big)\frac{\partial i_1}{\partial m_1} + f''(i_1)\frac{\partial i_1}{\partial m_1}\Delta a^*$$

implying that $\Delta a^* > 0$ is the sufficient condition for$\partial TW_1/\partial S^{SL} < 0$.

Since $\Delta a^* = f'(i_1)(i_1 - (\rho_1^* - d_0^* + S^{SL}) - \frac{1}{f'(i_1)}\phi^*)$, the necessary and sufficient condition for $\Delta a^* > 0$ is

$$(\rho_1 - d_0) + \frac{1}{f'(i_1)}(\phi - \gamma\phi^*) > (\rho_1^* - d_0^*) + \frac{1}{f'(i_1)}\phi^*$$