Optimal Central Bank Swap Line

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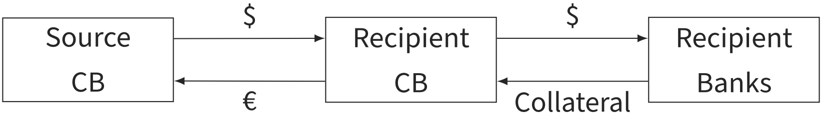
July 24, 2024

Motivation

Central Bank Swap Lines: providing currency of the source central bank (Fed) to the recipient central bank (ECB)



⇒ Lender of last resort: collateralized public liquidity line

- Implemented during financial distress (e.g. GFC, pandemic) ▸ chart ▸ summary
- Ceiling on CIP deviations (Bahaj & Reis, 2022): mitigates distress in international financial markets

What is the optimal policy? (Bagehot, 1873): Understudied

Research Question

1. What is the trade-off of the swap line policy?
2. What is the optimal swap line policy?
3. Can we improve by combining with other macroprudential policies?

Key Takeaway

- Ex-post optimal swap line (discretion): beneficial to both countries
  - Beneficial to recipient country: relieves stress in synthetic dollar funding market
  - Beneficial to source country: mitigates spillback due to integrated asset markets
  - Zero liquidity provision during normal times
- Ex-ante optimal swap line (commitment): lower liquidity provision
  - Overborrowing due to pecuniary externality
  - Swap line induces larger ex-ante borrowing
- Policy mix with macroprudential policy: constrained efficient
  - Discretion = Commitment
  - Tax on ex-ante borrowing corrects pecuniary externality

Literature Review

Central bank swap lines

- Effects of swap line policy: Baba & Parker (2009a,b); Bahaj & Reis (2022a,b, 2023); Kekre & Lenel (2023)

⇒ What I do: optimality of the swap line policy (normative analysis)

Optimal liquidity lines

- Pecuniary externality: Lorenzoni(2008); Jeanne & Korinek (2020); Schmitt-Grohé and Uribe (2021)
- Aggregate demand externality: Bianchi (2016); Farhi & Werning (2016); Korinek & Simsek (2016)
- Collective moral hazard: Farhi & Tirole (2012)

⇒ What I do: extension to an international setting focusing on the pecuniary externality

Model

Environment

Extension of Jeanne & Korinek (2020) to an international setting

- 3-period: $t = 0, 1, 2$
- Two countries: US (source) and EU (recipient) with measure 1/2
  - EU variables: denoted with asterisk (*)
  - Numeraire of US (EU): US (EU) goods, denoted as \$ (€)
- Agents: bankers (b) and depositors (d) with measure 1 in each country
  - Consume only at $t = 2$
  - Utility: linear in consumption
- Only one asset: US asset
  - Both the US banks and EU banks invest in US assets
  - Depositors can't invest ($\because$ can't use assets)
- Deposit rates: zero
- Exchange rates: spot (e) and forward (f), expressed in €per \$

US Bank

Period 0:

- Source of funds: endowed with exogenous ρ_0, issue deposits d_0,
- Use of funds: invests i_0

$$i_0 = \rho_0 + d_0$$

$\Rightarrow$ Assets at the end of period $t = 0$: $a_0 = f(i_0)$

- $f(\cdot)$: production function of assets, $f' > 0$, $f'' < 0$
- One unit of asset delivers one unit of payoff at $t = 2$

US Bank

Period 1:

- Source of funds: endowed with exogenous ρ_1, issue deposits d_1, sell Δa of assets at price p
  - ρ_1: source of uncertainty, realized at the beginning of $t = 1$
- Use of funds: invests i_1, trades FX swap S_1, repays deposits d_0
  - \$ S_1: exchange with €$e_1 S_1 \Rightarrow$ hold €$e_1 S_1$ ▸ FX swap
- Key financial frictions:
  1. Margin requirement: set aside γ fraction of S (Ivashina et al., 2015)
  2. Limited commitment: ϕ units of assets as collateral
     ★ No constraint on S_1 ($\because$ cash-like)

$$i_1 + (1+\gamma)S_1 + d_0 = \rho_1 + d_1 + p_1 \Delta a$$

$$d_1 - S_1 \leq p_1 \phi \qquad \text{(CC)}$$

US Bank

Period 2

- Source of funds: total assets, returns from S, set aside margin γS
  - Assets at the end of period $t = 1$: $a_1 = f(i_0) - \Delta a + f(i_1)$
  - Returns from S: return €$e_1 S_1$ that they hold and get \$ $(e_1/f_1)S_1$
    - ★ $e_1/f_1 - 1 \equiv \chi_1$: CIP deviations due to zero deposit rates
- Use of funds: consume c_2^b and repay deposits d_1

$$c_2^b + d_1 = f(i_0) - \Delta a + f(i_1) + \underbrace{\frac{e_1}{f_1}}_{=1+\chi_1} S_1 + \gamma S_1$$

$\Rightarrow$ Objective function: $c_2^b = f(\rho_0 + d_0) + \rho_1 - d_0 + f(i_1) - i_1 - (1 - p_1)\Delta a + \chi_1 S_1$

- Choice variables: $d_0, i_1, \Delta a, S_1$
- Exogenous variables: ρ_0, ρ_1

EU Bank

Period 0: $i_0^* = \rho_0^* + d_0^*$ and $a_0^* = f(i_0^*)$

- All variables are denominated in \$
- Assumption: EU banks *can* issue deposits in \$ from US depositors

Period 1:

- Use of funds: invests i_1^*, repays deposits d_0^*
- Source of funds: $\rho_1^*, d_1^*, \Delta a^*$ sold at price p
  - Assumption: EU banks *cannot* issue deposits in \$ at $t = 1$
  - Motivation: dry-up of direct dollar funding $\Rightarrow$ Need to borrow in €and convert to \$ using FX swap (synthetic dollar funding)

$$i_1^* + d_0^* = \rho_1^* + \frac{1}{e_1} d_1^* + p_1 \Delta a^*$$

- Limited commitment constraint with collateral ϕ units of assets: $\frac{1}{e_1} d_1^* \leq p_1 \phi^*$

EU Bank

Period 2

- EU banks consume US goods c_2^{b*}
- Get €d_1^* from US banks, give \$$d_1^*/f_1$ ‣ FX swap

$$c_2^{b*} + \frac{1}{e_2} d_1^* = f(i_0^*) - \Delta a^* + f(i_1^*) + \frac{1}{e_2} d_1^* - \frac{1}{f_1} d_1^*$$

⇒ Objective function:

$$c_2^{b*} = f(\rho_0^* + d_0^*) + (1 + \chi_1)(\rho_1^* - d_0^*) + f(i_1^*) - (1 + \chi_1) i_1^* - (1 - (1 + \chi_1) p_1) \Delta a^*$$

- Effective cost of investment: $1 + \chi_1$
  - χ_1: costs of currency matching
- Choice variables: $d_0^*, i_1^*, \Delta a^*$
- Exogenous variables: ρ_0^*, ρ_1^*

Depositors and Market Clearing

Depositors

- Endowment y and y^* for US and EU depositors in period 0 and 1
- Saving or storing output: return rate = 0
- Can't use asset = 0 $\Rightarrow$ do not trade assets
- US depositor's consumption (in \$):
  $c_2^d = (y - d_0 - d_0^*) + (y + d_0 + d_0^* - d_1) + d_1 = 2y$
- EU depositor's consumption (in €): $c_2^{d*} = y^* + (y^* - d_1^*) + d_1^* = 2y^*$

Market clearing conditions:

- Asset market: $\Delta a + \Delta a^* = 0$
- FX swap market: $S_1 = d_1^*/e_1$

First-Best Allocation

Definition

The first-best allocation is defined by the allocation solving the social planner problem without any financial friction

$$\max_{d_0, d_0^*, i_1, i_1^*, \Delta a, \Delta a^*} E\left[\frac{1}{2}(c_2^b + c_2^d) + \frac{1}{2}(c_2^{b*} + c_2^{d*})\right]$$

$$= \frac{1}{2}E\Big[f(\rho_0 + d_0) + \rho_1 - d_0 + f(i_1) - i_1 + 2y$$

$$+ f(\rho_0^* + d_0^*) + \rho_1^* - d_0^* + f(i_1^*) - i_1^* + 2y^*\Big]$$

Proposition

The first-best allocation is characterized by $i_1 = i_1^* = i_1^{FB}$ *satisfying* $f'(i_1^{FB}) = 1$ *and* $d_0 = d_0^{FB}$, $d_0^* = d_0^{*FB}$ *satisfying* $f'(\rho_0 + d_0^{FB}) = f'(\rho_0^* + d_0^{*FB}) = 1$. ▸ proof

Optimal Swap Line Policy

Swap Line Policy

Policy instrument: state-contingent \$ provision S^{SL}

- In practice, swap spread ss
  - $\chi_1 \leq ss$: ceiling on CIP deviations (Bahaj & Reis, 2022)
- 1-1 relation between S^{SL} and $\bar{\chi}$
  - $S^{SL} \geq 0$ fills excess demand for synthetic dollar funding

Implementation

- At $t = 1$, US government borrows S^{SL} from US depositors and lends the same amount to EU banks
- At $t = 2$, it gets repayment of $(1 + \chi_1)S^{SL}$, repays S^{SL} to depositors, and net return $\chi_1 S^{SL}$ is rebated to US banks.
- $L(S^{SL})$: deadweight loss from S^{SL} where $L' > 0$, $L'' > 0$, $L(0) = L'(0) = 0$

▸ endegeneize

S^{SL}: What Does It Change?

US bank: $t = 2$ budget constraint changes:

$$c_2^b + d_1 = f(i_0) - \Delta a + f(i_1) + (1+\chi_1)S_1 + \gamma S_1 + \chi_1 S^{SL}$$

- $\chi_1 S^{SL}$: rebates of net returns from S^{SL}

EU bank: $t = 1$, $t = 2$ budget constraints change:

$$i_1^* + d_0^* = \rho_1^* + \frac{1}{e_1} d_1^* + p_1 \Delta a^* + S^{SL}$$

$$c_2^{b*} + \frac{1}{e_2} d_1^* = f(i_0^*) - \Delta a^* + f(i_1^*) + \frac{1}{e_2} d_1^* - \frac{1}{f_1} d_1^* - (1+\chi) S^{SL}$$

- Borrow S^{SL} and repay $(1+\chi_1)S^{SL}$

US depositor: $c_2^d = 2y - L(S^{SL})$

Optimization at $t = 1$: US Bank

Ignoring predetermined $f(\rho_0 + d_0) + \rho_1 - d_0$,

$$V_1^b(\rho_1 - d_0) \equiv \max_{i_1, \Delta a, S_1} f(i_1) - i_1 - (1 - p_1)\Delta a + \chi_1 S_1 + \chi_1 S^{SL}$$

$$\text{s.t. } i_1 \leq (\rho_1 - d_0) + p_1(\phi + \Delta a) - \gamma S_1$$

- First-order conditions: For the Lagrangian multiplier $\lambda_1 \geq 0$ of CC,

$$f'(i_1) = 1 + \lambda_1$$

$$p_1 = 1/f'(i_1)$$

$$\chi_1 = \gamma(f'(i_1) - 1)$$

$$\lambda_1(\rho_1 - d_0 + p_1(\phi + \Delta a) - \gamma S_1 - i_1) = 0$$

Optimization at $t = 1$: EU Bank

$$V_1^{b*}(\rho_1^* - d_0^*) \equiv \max_{i_1, \Delta a^*} f(i_1^*) - (1+\chi_1)i_1^* - (1 - (1+\chi_1)p_1)\Delta a^*$$

$$\text{s.t. } i_1^* \leq \rho_1^* - d_0^* + p_1(\phi^* + \Delta a^*) + S^{SL}$$

- First-order conditions: For the Lagrangian multiplier $\lambda_1^* \geq 0$ of CC,

$$f'(i_1^*) = 1 + \chi_1 + \lambda_1^*$$

$$p = 1/f'(i^*)$$

$$\lambda_1^*(\rho_1^* - d_0^* + p_1(\phi^* + \Delta a^*) + S^{SL} - i_1^*) = 0$$

  - Asset markets are integrated: $f'(i_1) = 1/p = f'(i_1^*) \Rightarrow i_1 = i_1^*$
  - $\lambda_1^* = (1-\gamma)\lambda_1 \Rightarrow \lambda_1 > 0$ iff $\lambda_1^* > 0$

Competitive Equilibrium at $t = 1$

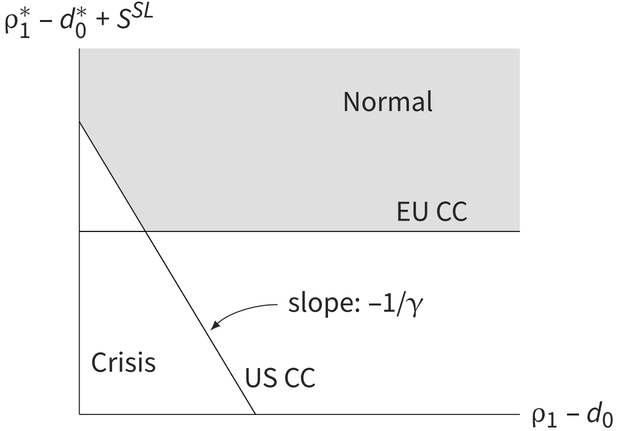
Two regimes in the EU: normal ($\lambda_1^* = 0$) vs crisis ($\lambda_1^* > 0$)

- $\lambda_1^* = 0$: $i_1 = i_1^* = i_1^{FB}$, $p_1 = 1$, $\chi_1 = 0$ since $f'(i_1) = f'(i_1^*) = 1$
  - Condition: collateral constraints do not bind ▸ graph ▸ conditions
- $\lambda_1^* > 0$: i_1 is determined by the fixed point of

$$i_1 = \underbrace{\frac{1}{2}\left[(\rho_1 - d_0) + (\rho_1^* - d_0^*) + S^{SL} + \frac{1}{f'(i_1)}\left(\phi + (1-\gamma)\phi^*\right)\right]}_{G(i_1; m_1)}$$

  - Does i_1 uniquely exist? ▸ proposition
  - i_1: function of available dollar $m_1 \equiv (\rho_1 - d_0) + (\rho_1^* - d_0^*) + S^{SL}$
  - $p_1 < 1$ and $\chi_1 > 0$

Conditions: Normal vs Crisis Regime



- As ρ_1^* declines, ρ_1 needs to be higher for the normal regime
- If ρ_1^* is below a threshold, the crisis regime is always realized ▸ back

Existence & Uniqueness of the Ex-Post Equilibrium

Lemma

i_1 in the crisis regime uniquely exists if and only if for all $i_1 < i_1^{FB}$ there exists $c < 1$ such that

$$G'(i_1) = \underbrace{-\frac{f''(i_1)}{f'(i_1)^2}}_{=\partial p_1/\partial i_1} \frac{1}{2}(\phi + (1-\gamma)\phi^*) \leq c$$

- $G'(i_1) > 0$ since $f'' < 0$: pecuniary externality
- $G'(i_1) = 0$ if there is no pecuniary externality

▸ back

Effect of Swap Line on Investments

Proposition

In the crisis regime,

$$\frac{\partial i_1}{\partial S^{SL}} = \frac{\partial i_1^*}{\partial S^{SL}} = \frac{1}{2}\frac{1}{1 - G'(i_1)}$$

which is strictly positive if and only if i_1 uniquely exists.

Financial accelerator: pecuniary externality

- Banks don't internalize the effect of i_1 on $p_1 = 1/f'(i_1)$
  - As $i_1 \uparrow$, $p_1\phi$ and $p_1\phi^* \uparrow$, relaxing US and EU CC
  - As $i_1 \uparrow$, $S_1 = p_1\phi^* \uparrow$, tightening US CC by a factor of γ
- Since $G'(i_1) > 0$ when there is pecuniary externality, $\partial i_1/\partial S^{SL} > 1/2$
  - $\partial i_1/\partial S^{SL} = 1/2$ without pecuniary externality
- $\partial \chi_1/\partial S^{SL} < 0$ since $\chi_1 = \gamma(f'(i_1) - 1)$

Discretion: Ex-Post Optimal Policy

Discretion policy: ex-post efficient policy (after realizations of regimes)

Ramsey problem:

$$W_1(m) \equiv \max_{S^{SL}} \frac{1}{2}[f(i_1(m)) - i_1(m) - L(S^{SL})] + \frac{1}{2}[f(i_1^*(m)) - i_1^*(m)]$$

- Normal regime: $S^{SL} = 0$ since $i_1 = i_1^* = i_1^{FB}$
- Crisis regime:

$$\underbrace{(f'(i_1) - 1)}_{\lambda_1}\frac{\partial i_1}{\partial S^{SL}} + \underbrace{(f'(i_1^*) - 1)}_{\chi_1 + \lambda_1^*}\frac{\partial i_1^*}{\partial S^{SL}} = \frac{\lambda_1}{1 - G'(i_1)} = L'(S^{SL})$$

  - $S^{SL} > 0$ since $L'(0) = 0$
  - $i_1 = i_1^* < i_1^{FB}$: partial liquidity provision due to deadweight loss
  - Pecuniary externality amplifies benefits of swap line policy

Optimization at $t = 0$

US bank: For the ex-post value function $V_1^b(\rho_1 - d_0)$,

$$\max_{d_0} E\left[f(\rho_0 - d_0) + \rho_1 - d_0 + V_1^b(\rho_1 - d_0)\right]$$

- $V_1^{b'}(\rho_1 - d_0) = \lambda_1$ by envelope condition
- Optimality condition: $f'(\rho_0 - d_0) = 1 + E[\lambda_1]$
  - $E[\lambda_1]$: expected shadow cost due to the marginal change in d

EU bank: For the ex-post value function $V_1^{b*}(\rho_1^* - d_0^*)$,

$$\max_{d_0^*} E\left[f(\rho_0^* + d_0^*) + (1 + \chi_1)(\rho_1^* - d_0^*) + V_1^{b*}(\rho_1^* - d_0^*)\right]$$

- Optimality condition: $f'(\rho_0^* + d_0^*) = 1 + E[\chi_1 + \lambda_1^*] = 1 + E[\lambda_1]$

Overborrowing

Social planner problem: For the ex-post global value function $W(\cdot, \cdot)$,

$$\max_{d_0, d_0^*} E\left[\frac{1}{2}\left\{f(\rho_0 + d_0) + \rho_1 - d_0\right\} + \frac{1}{2}\left\{f(\rho_0^* + d_0^*) + \rho_1^* - d_0^*\right\} + W_1(m)\right]$$

- Optimality conditions:

$$f'(\rho_0 + d_0) = 1 + E\left[\frac{\lambda_1}{1 - G'(i_1)}\right] > 1 + E[\lambda_1]$$

$$f'(\rho_0^* + d_0^*) = 1 + E\left[\frac{\chi_1 + \lambda_1^*}{1 - G'(i_1)}\right] > 1 + E[\chi_1 + \lambda_1^*]$$

- Implication: overborrowing since $G'(i_1) > 0$
  - Social planner takes effects of d_0 and d_0^* on the asset price into account

Commitment

Commitment: ex-ante efficient policy considering overborrowing

Ramsey problem:

$$\max_{d_0, d_0^*, S^{SL}} E\left[\frac{1}{2}\left\{f(\rho_0 + d_0) + \rho_1 - d_0 + f(i_1(m)) - i_1(m) - L(S^{SL})\right\}\right.$$
$$\left. + \frac{1}{2}\left\{f(\rho_0^* + d_0^*) + \rho_1^* - d_0^* + f(i_1^*(m)) - i_1^*(m)\right\}\right]$$
$$\text{s.t. } f'(\rho_0 + d_0) = 1 + E[\lambda_1]$$
$$f'(\rho_0^* + d_0^*) = 1 + E[\lambda_1]$$

- Optimality condition for S^{SL}: For Lagrangian multipliers ν_1 and ν_1^*,

$$\frac{\lambda_1}{1 - G'(i_1)} + \underbrace{(\nu_1 + \nu_1^*)\frac{f''(i_1)}{1 - G'(i_1)}}_{\text{cost of overborrowing: } (\nu_1 + \nu_1^*)\partial\lambda/\partial S^{SL}} = L'(S^{SL}) < \frac{\lambda_1}{1 - G'(i_1)}$$

- Lower S^{SL} under commitment than discretion

Policy Mix: Macroprudential Policy

Tax on d_0 and d_0^* by each government: τ^d and τ^{d*}

- Tax revenues are rebated to banks in each jurisdiction

US bank:

$$\max_{d_0} E\left[f(\rho + (1 - \tau^d)d_0 + \tau^d \tilde{d}_0) + \rho_1 - d_0 + V_1^b(\rho_1 - d_0)\right]$$

- $\tilde{d}_0$: aggregate deposits (exogenous to individual bank)
- FOC: $(1 - \tau^d)f'(\rho + d_0) = 1 + E[\lambda_1]$

EU bank:

$$\max_{d_0^*} E\left[f(\rho_0^* + (1 - \tau^{d*})d_0^* + \tau^{d*}\tilde{d}_0^*) + (1 + \chi_1)(\rho_1^* - d_0^*) + V_1^{b*}(\rho_1^* - d_0^*)\right]$$

- FOC: $(1 - \tau^{d*})f'(\rho_0^* + d_0^*) = 1 + E[\lambda_1]$

Optimal Policy Mix

Optimal tax to achieve constrained efficient ex-ante borrowing:

$$\tau^d = \tau^{d*} = \frac{E\left[\left(\frac{1}{1-G'(i_1)} - 1\right)\lambda_1\right]}{f'(\rho_0 + d_0)} > 0$$

- Corrects pecuniary externality
- Achieves constrained efficiency
- Discretion = Commitment ‣ proof
  - Potential threat to the ex-ante financial stability is dealt with macroprudential policies

Cooperative Ramsey Problem

Question: does a global Ramsey planner exist in the real world?

$\Rightarrow$ Cooperative Ramsey problem at $t = 1$: (Benigno & Benigno, 2003)

$$\max_{S^{SL}} \alpha \Big[\underbrace{f(i_1(m_1)) - i_1(m_1) - L(S^{SL}) + TW_1}_{V_1(m_1)\,:\,\text{US welfare}} \Big] + (1-\alpha) \Big[\underbrace{f(i_1^*(m)) - i_1^*(m) - TW_1}_{V_1^*(m_1)\,:\,\text{EU welfare}} \Big]$$

- $TW_1 \equiv \chi_1(m)(S_1(m) + S^{SL}) + (1 - p_1(m_1))\Delta a^*(m)$: transfer of wealth to US
- α: bargaining power of US
- Global Ramsey planner: special case with $\alpha = 1/2$
  - Transfer of wealth is cancelled

Policy Coordination

Optimality condition:

$$\frac{1}{2}\underbrace{\left[\frac{\lambda_1}{1-G'(i_1)} - L'(S^{SL})\right]}_{=0 \text{ under global Ramsey problem}} + (2\alpha - 1)\left[\frac{\partial TW}{\partial m_1} - \frac{1}{2}L'(S^{SL})\right] = 0$$

Assumption: $\partial TW_1/\partial m_1 < 0$ ▸ proof

- Sufficient condition: $\Delta a^* > 0$, *i.e.* EU sells assets to US during crisis
- Equivalent to $(\rho_1 - d_0) + (\phi - \gamma\phi^*)/f'(i_1) > (\rho_1^* - d_0^*) + \phi^*/f'(i_1)$

Implication

- $\alpha > 1/2$: S^{SL} is lower than the global Ramsey solution (undersupply)
- $\alpha < 1/2$: S^{SL} is larger than the global Ramsey solution (oversupply)

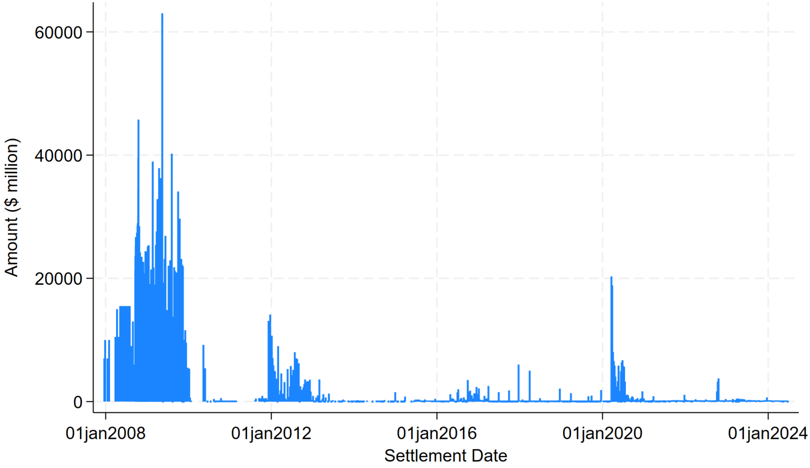
Conclusion

Future Work

- Analyze defaults and its consequences
  - Collateral value during defaults?
  - Related to balance of payment crisis?
- Extend to a quantitative & dynamic model
  - Evaluate the current swap line policy

Appendix

Fed Swap Line: Transaction



Summary Statistics of Fed Swap Line Transaction

Sample (settlement date): 12/20/2007 - 6/21/2024

Maturity:

Maturity	Overnight	1-week	2-week	3-week	4-week	5-week	12-week	13-week
Obs	165	1230	39	15	107	7	288	10
Mean ($ mil)	20221	4576	3056	6320	10604	5906	4672	4688

Recipient Currency:

Currency	AUD	CHF	DKK	EUR	GBP	JPY	KRW	MXN	NOK	SEK	SGD
Obs	14	209	28	971	137	402	28	17	12	10	39
Mean ($ mil)	3882	2584	2884	9087	7379	2590	2188	1465	2925	6720	632

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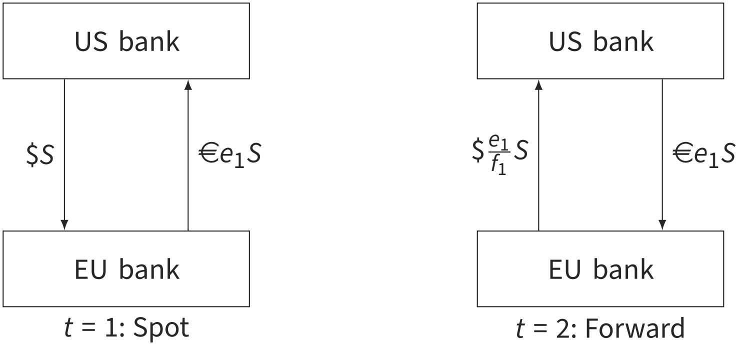
Structure of a FX Swap Contract



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Proof: First-Best Allocation

Unconstrained optimization problem:

$$\max_{i_1, i_1^*, d_0, d_0^*} E\left[\frac{1}{2}(c_2^b + c_2^d) + \frac{1}{2}(c_2^{b*} + c_2^{d*})\right]$$

$$= \frac{1}{2}E\Big[f(\rho_0 + d_0) + \rho_1 - d_0 + f(i_1) - i_1 + 2y$$

$$+ f(\rho_0^* + d_0^*) + \rho_1^* - d_0^* + f(i_1^*) - i_1^* + 2y^*\Big]$$

First-order conditions:

$$f'(i_1) = f'(i_1^*) = 1$$

$$f'(\rho_0 + d_0) = f'(\rho_0^* + d_0^*) = 1$$

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Overview of Central Bank Swap Lines

Swap line spread: ceiling on CIP deviations (Bahaj & Reis, 2022)

$$cid \leq ss + (r^{p*} - r^{\nu *})$$

- Due to no-arbitrage condition regarding lending from the central bank
  - Cost of swap lines: $r^{OIS} + ss$
  - Cost of synthetic dollar funding: $r^{OIS} + cid + r^{\nu *} - r^{p*}$
- International version of the domestic discount window

Conditions for the Normal Regime

Non-binding collateral constraints at $i_1 = i_1^* = i_1^{FB}$:

$$i_1^{FB} < \rho_1 - d_0 + \phi - \gamma(i_1^{FB} - (\rho_1^* - d_0^*) - S^{SL})$$
$$i_1^{FB} < \rho_1^* - d_0^* + \phi^* + S^{SL}$$

$\Rightarrow$ Conditions for the normal regime in $\rho_1 - d_0$ and $\rho_1 - d_0^* + S^{SL}$:

$$(\rho_1 - d_0) + \gamma(\rho_1 - d_0^* + S^{SL}) > (1+\gamma)i^{FB} - \phi$$
$$\rho_1 - d_0^* + S^{SL} > i^{FB} - \phi^*$$

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Endogeneizing Deadweight Loss

Depositors: produce y from labor h with disutility $C(h)$ for $C', C'' > 0$

- Optimization problem: $\max_h y \equiv h - C(h)$
- First-best solution: $C'(h^{FB}) = 1$ and $y^{FB} = h^{FB} - C(h^{FB})$

Swap line: fund S^{SL} with tax τ^s on sales

- $\max_h (1 - \tau^s)h - C(h) \Rightarrow C'(h) = 1 - \tau^s \Rightarrow h = h(\tau^s) \equiv (C')^{-1}(1 - \tau^s)$
- $S^{SL} = \tau^s h(\tau^s) \Rightarrow \tau^s = \tau^s(S^{SL})$
- $y(S^{SL}) = h(\tau^s(S^{SL})) - C(h(\tau^s(S^{SL})))$

$\Rightarrow$ Deadweight loss:

$$c_d = y^{FB} + y(S^{SL}) = 2y^{FB} - \underbrace{(y^{FB} - y(S^{SL}))}_{\equiv L(S^{SL})}$$

▸ back

Proof: S^{SL} is lower under commitment

First-order conditions for Ramsey problem:

$$E\left[\frac{1}{2}\left(f'(n+d)-1-\frac{\lambda}{1-G'(i)}\right)-\nu\left(\frac{1}{2}\frac{f''(i)}{1-G'(i)}+f''(n+d)\right)-\nu^*\frac{1}{2}\frac{f''(i)}{1-G'(i)}\right]=0$$

$$E\left[\frac{1}{2}\left(f'(n+d)-1-\frac{\lambda}{1-G'(i)}\right)-\nu\frac{1}{2}\frac{f''(i)}{1-G'(i)}-\nu^*\left(\frac{1}{2}\frac{f''(i)}{1-G'(i)}+f''(n+d)\right)\right]=0$$

$$\frac{\lambda}{1-G'(i)}+(\nu+\nu^*)\frac{f''(i)}{1-G'(i)}=L'(S^{SL})$$

$\Rightarrow$ From the first two equations, since $G'(i)>0$

$$\nu+\nu^*=\frac{E\left[\left(1-\frac{1}{1-G'(i)}\right)\lambda\right]}{E[\frac{f''(i)}{1-G'(i)}]+f''(n+d)}>0$$

▸ back

Proof: Constrained Efficiency of Optimal Policy Mix

Ramsey problem:

$$\max_{d,d^*,S^{SL}} E\left[\frac{1}{2}\left\{f(n+d)+\rho-d+f(i(m))-i(m)-L(S^{SL})+f(n^*+d^*)+\rho^*-d^*+f(i^*(m))-i^*(m)\right\}\right]$$

$$\text{s.t.}\quad (1-\tau)f'(n+d)=1+E[\lambda]$$

$$(1-\tau^*)f'(n^*+d^*)=1+E[\lambda]$$

- Optimality conditions without implementability conditions:

$$f'(n+d)=1+E\left[\frac{\lambda}{1-G'(i)}\right]$$

$$f'(n^*+d^*)=1+E\left[\frac{\lambda}{1-G'(i)}\right]$$

$$\frac{\lambda}{1-G'(i)}=L'(S^{SL})$$

- Implementability conditions: satisfied by optimal tax τ and τ^* ▸ back

Sufficient Condition for $\partial \Delta W / \partial S^{SL} < 0$

Sufficient condition: $\Delta k^* > 0$

$$\frac{\partial \Delta W}{\partial S^{SL}} = -(1-\gamma)(f'(i)-1) + \gamma f''(i)\left(\frac{\phi^*}{f'(i)^2} + S^{SL}\right)\frac{\partial i}{\partial m}$$
$$+ f''(i)\frac{\partial i}{\partial m}\left(i - (\rho^* - d^* - \frac{\phi^*}{f'(i)^2} - S^{SL}\right)$$
$$< -(1-\gamma)(f'(i)-1) + \gamma f''(i)\left(\frac{\phi^*}{f'(i)^2} + S^{SL}\right)\frac{\partial i}{\partial m} + f''(i)\frac{\partial i}{\partial m}\Delta k^* < 0$$

Necessary and sufficient condition for $\Delta k^* > 0$:

$$\underbrace{(\rho - d)}_{\text{available \$ in US}} + \underbrace{\frac{1}{f'(i)}(\phi - \gamma\phi^*)}_{\text{\$ funding net of haircut}} > \underbrace{(\rho^* - d^*)}_{\text{available \$ in EU}} + \underbrace{\frac{1}{f'(i)}\phi^*}_{\text{synthetic \$ funding}}$$

- \$ is more ample in the US back